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On Finding Approximate Solutions to Nonlinear PDEs Using the ADM and DTM

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Abstract In this paper a new approach for finding approximate solutions using the Differential Transform Method (DTM) and the Adomian Decomposition Method (ADM) to two different types of nonlinear partial differential equations (PDEs), such as; general Korteweg-de Vries equation (GKdV) and general improved Korteweg-de Vries equation (GIKdV). A numerical approximation is obtained with excellent accuracy and comparison shows the DTM and ADM are easy and efficient to use.

MSC: 34K28, 35G25, 34K17

Keywords: Differential Transform Method, Adomian Decomposition Method, GKdV, GIKdV, partial differential equations, Numerical solutions.

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1. Introduction

The concept of Differential Transform Method (DTM) was introduced by Pukhov 1984 [9,10,11], who solved linear and non-linear initial problems in electric circuit analysis. This method is frequently presented as a (relatively) new method for solving differential equations. Though based on Taylor series, it would be different from the traditional Taylor (or power) series method presented in usual textbooks. The differential transformation technique is one of the numerical methods for ordinary differential equations. It uses the form of polynomials as the approximation to exact solutions which are sufficiently differentiable. This is in contrast to the traditional high-order Taylor series method. This method is useful to obtain exact and approximate solutions of linear and non-linear differential equations.

The Adomian decomposition method (ADM), see [1-5], proposed by George Adomian, has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear. The ADM, which accurately computes the series solution, is of great interest to applied sciences. The method provides the solution in a rapidly convergent series with components that are elegantly computed. The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear,

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homogeneous or inhomogeneous, with constant coefficients or with variable coefficients. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution.

The aim of our study is to introduce the DTM and ADM as an alternative to existing methods in solving different types of nonlinear PDEs such as: The general Korteweg−de Vries Equation (GKdV) in the form, see [6]

$$u_t + \epsilon u^p u_x + \gamma u_{xxx} = 0, \tag{1.1}$$

where p is a positive integer, ϵ and ν are positive constants which require the boundary conditions $u \to 0$ as $x \to \pm\infty$, and x is the space variable, t is the time. The general improved Korteweg−de Vries equation (GIKdV) has the form, see [6]

$$u_t + \epsilon u^p u_x + \gamma u_{xxx} - \nu u_{xxt} = 0. \tag{1.2}$$

The rest of this paper is organized as follows: In Section 2 and 3, the DTM and ADM are introduced. Section 4 is devoted to apply the DTM and ADM to two test problems to show the effectiveness of both methods. Section 5 discussion and conclusion of this paper.

2. BASIC DEFINITIONS AND BACKGROUND OF THE DTM AND ADM

The basic definitions and fundamental operations of the one-dimensional differential transform are defined as follows, see [10]: Let $f(x)$ be an analytic function in the real numbers, and let x_0 be a real number. The function $f(x)$ is then represented by one series whose center is located at x_0. The differential transform of the function $f(x)$ is of the form:

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=x_0}, \tag{2.1}$$

and the differential transform inverse of $F(k)$ is defined as

$$f(x) = \sum_{k=0}^{\infty} F(k)(x - x_0)^k. \tag{2.2}$$

In real applications, the function $f(x)$ is expressed by a finite series and Equation (2.2) can be written as:

$$f(x) = \sum_{k=0}^{N} F(k)(x - x_0)^k. \tag{2.3}$$

Note that from the above discussion, one can realize that the DTM is derived from the power series expansion. Some basic operations of the differential transformation obtained from equations (2.1) and (2.2) are given in the table below:

Table 1. Basic operations of the DTM [9–11]

Functional Form	**Transformed form**
f(x)=g(x)± h(x)	F(k)=G(k)±H(k)
f(x)=α g(x)	F(k)=α G(k), where α is a constant.
f(x)=$\frac{d^n g(x)}{dx^n}$	F(k)=$\frac{(k+n)!}{k!}G(k+n)$.
f(x)=g(x).h(x)	$F(k)=\sum_{i=0}^{k} G(i).H(k-i)$.
f(x)=x^n	F(k)=δ(k-n), where δ(k-n)= $\begin{cases} 1, & \text{n=k;} \\ 0, & \text{n}\neq\text{k.} \end{cases}$
f(x)=u(x).v(x).w(x).	F(k)=$\sum_{i=0}^{k}\sum_{j=0}^{i} U(j)\,V(i-j)\,W(k-i)$
f(x)=$\int_{x_0}^{x}$h(t)dt	F(k)=$\frac{H(k-1)}{k}$, where $k \geq 1$.

Methodology of the DTM.

Now, we illustrate the DTM by using the GKdV equation in standard form with $(p = 1)$:

$$u_t + \epsilon\, u u_x + \gamma\, u_{xxx} = 0, \tag{2.4}$$

subject to the initial condition

$$u(x, 0) = \frac{3c}{\epsilon}\, sech^2\left(\frac{\sqrt{c}x}{2\sqrt{\gamma}}\right), \tag{2.5}$$

where x is the space variable, t is the time.

Now we transform the nonlinear of equation (2.4) into nonlinear ODE by letting $\xi = x-ct$, where c is the velocity of the wave such that $u(x, t) = u(\xi)$ is the solution of equation (2.4). Now equation (2.4) becomes

$$-c\, u'(\xi) + \epsilon\, u(\xi)u'(\xi) + \gamma\, u'''(\xi) = 0. \tag{2.6}$$

Now integrate Equation (2.6) with respect to ξ to get

$$-c\, u(\xi) + \frac{\epsilon}{2}u^2(\xi) + \gamma\, u''(\xi) = 0. \tag{2.7}$$

Applying the differential transform to equation (2.7), and using Table 1, we get

$$\gamma\,(k + 1)(k + 2)U(k + 2) - c\, U(k) + \frac{\epsilon}{2}\sum_{i=0}^{k} U(i)U(k - i) = 0, \tag{2.8}$$

which is equivalent to

$$U(k + 2) = \frac{c\, U(k)}{\gamma\,(k + 1)(k + 2)} - \frac{\epsilon}{2\gamma\,(k + 1)(k + 2)}\sum_{i=0}^{k} U(i)U(k - i) \tag{2.9}$$

where $U(k)$ represent the DT of $u(\xi)$, $k \geq 0$, and $U(0)$ and $U(1)$ are:

$$U(0) = \frac{1}{0!}\left[\frac{d^0 u(x)}{dx^0}\right]_{x=0} = u(0) = \frac{3c}{\epsilon}, \tag{2.10}$$

and

$$U(1) = \frac{1}{1!}\left[\frac{du(x)}{dx}\right]_{x=0} = u'(0). \tag{2.11}$$

We set $U(1) = \beta$, and starting with $U(0)$ and $U(1)$, then $U(2)$ can be obtained from equation (2.9). By using $U(0)$, $U(1)$ and $U(2)$, $U(3)$ can be determined easily. Continuing in this manner, the N- differential transforms of $U(\xi)$ can be identified. These differential transforms depend on the variable ξ, and the constants c and β.
Now, applying the inverse transform of $U(k)$ using equation (2.9) we get,

$$u(\xi) = \sum_{k=0}^{N} U(k)\xi^k. \tag{2.12}$$

Finally, the constants c and β will be determined using $u(x,0) = \frac{3c}{\epsilon}\, sech^2\left(\frac{\sqrt{c}x}{2\sqrt{\gamma}}\right)$ for different values of x.

Methodology of the ADM.

It is well known that Adomian decomposition method suggests that the unknown linear function u may be represented by the decomposition series,

$$u = \sum_{n=0}^{\infty} u_n, \tag{2.13}$$

where the components u_n, $n \geq 0$ are to be determined in a recursive manner. However, the nonlinear terms $F(u)$, such as $u^2, u^3, u^4, sin(u), e^u, uu_x, u_x^2, etc.$ can be expressed by an infinite series of the so-called Adomian polynomials A_n given in the form,

$$F(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, u_2, ..., u_n), \tag{2.14}$$

where the A_n for the nonlinear term $F(u)$ can be evaluated by using the following expression,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, n = 0, 1, 2, ... \tag{2.15}$$

The general formula (2.15) can be simplified as follows: Assuming that the nonlinear function is F(u), therefore by using (2.15), the Adomian polynomials, see [3] are given by:

$$\begin{aligned}
&A_0 = F(u_0),\\
&A_1 = u_1 F'(u_0),\\
&A_2 = u_2 F'(u_0) + \tfrac{1}{2!}u_1^2 F''(u_0),\\
&A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \tfrac{1}{3!}u_1^3 F'''(u_0),\\
&A_4 = u_4 F'(u_0) + (\tfrac{1}{2!}u_2^2 + u_1 u_3)F''(u_0) + \tfrac{1}{2!}u_1^2 u_2 F'''(u_0) + \tfrac{1}{4!}u_1^4 F^{(4)}(u_0).
\end{aligned} \tag{2.16}$$

Other polynomials can be generated in a similar manner. Two important observations can be made here. First, A_0 depends only on u_0, A_1 depends only on u_0 and u_1, A_2

depends only on u_0, u_1 and u_2, and so on. Second, substituting (2.14) into (2.16) gives:

$$\begin{aligned} F(u) &= A_0 + A_1 + A_2 + A_3 + ... \\ &= F(u_0) + (u_1 + u_2 + u_3 + ...)F'(u_0) \\ &+ \frac{1}{2!}(u_1^2 + 2u_1u_2 + 2u_1u_3 + u_2^2 + ...)F''(u_0) + ... \\ &+ \frac{1}{3!}(u_1^3 + 3u_1^2u_2 + 3u_1^2u_3 + 6u_1u_2u_3 + ...)F'''(u_0) + ... \\ &= F(u_0) + (u - u_0)F'(u_0) + \frac{1}{2!}(u - u_0)^2F''(u_0) + ... \end{aligned}$$

The last expansion confirms the fact that the series in A_n polynomials is a Taylor series about a function u_0 and not about a point as is usually used. The Adomian polynomials given above in (2.16) clearly show that the sum of the subscripts of the components of u of each term of A_n is equal to n.

3. APPLICATIONS OF THE DTM

In this section we present approximate solutions for the GKdV and GIKdV for special values using the DTM and the ADM. Our results will be compared with the exact solutions.

3.1. SOLVING THE GKDV AND GIKDV USING THE DTM

First, we apply the DTM to two test problems mentioned above.

Example 1.

Consider the nonlinear GKdV in the form ($p = 1, \epsilon = 0.5, \gamma = 1$):

$$u_t + 0.5\, uu_x + u_{xxx} = 0, \tag{3.1}$$

subject to the initial condition

$$u(x, 0) = 3\, sech^2 \left(\frac{x}{2\sqrt{2}} \right), \tag{3.2}$$

where the exact solution is

$$u(x, t) = 3\, sech^2 \left(\frac{x - 0.5t}{2\sqrt{2}} \right). \tag{3.3}$$

Now we transform the nonlinear equation (3.1) into nonlinear ODE by letting $\xi = x - ct$, where c is the velocity of the wave such that $u(x, t) = u(\xi)$ is the solution of Equation (3.1). Now equation (3.1) becomes

$$-c\, u'(\xi) + 0.5\, u(\xi)u'(\xi) + u'''(\xi) = 0. \tag{3.4}$$

Now integrate equation (3.4) with respect to ξ to get

$$u''(\xi) - c\, u(\xi) + 0.25u^2(\xi) = 0, \tag{3.5}$$

subject to the initial condition

$$u(0) = 3. \tag{3.6}$$

Applying the differential transform to equations (3.5–3.6) and by means of Table 1, we obtain the following recursive formula

$$U(k+2) = \frac{cU(k)}{(k+1)(k+2)} - \frac{1}{4(k+1)(k+2)}\sum_{i=0}^{k} U(i)U(k-i), k \geq 0. \tag{3.7}$$

and

$$U(0) = u(0) = 3, \quad U(1) = u'(0) = \beta, \tag{3.8}$$

where β is a constant to be determined later. Using $U(0)$, $U(1)$ we coded (3.7) in Mathematica, and obtain the following results:

$$U(2) = \frac{12c-9}{8}, U(3) = \frac{4c^2 - 4\beta - 3c}{16}. \tag{3.9}$$

Continuing in this manner, the first 10-iterations can be identified eventually by using mathematica software. Hence, the approximate solution can be expressed as:

$$u_{appr}(x,t) = \sum_{i=0}^{9} U(i)(x-ct)^i. \tag{3.10}$$

Now, using the initial condition (3.2) and by the aid of Mathematica software, the constants c and β are

$$c = 0.5007035288840904, \qquad \beta = -0.00112398890532712832.$$

Substituting the values of c and β in equation (3.10), the approximate solution is

$$\begin{aligned} u_{appr}(x,t) \;=\; & 3 - 0.00112399(x-0.500704t) - 0.373945(x-0.500704t)^2 \\ & +0.0001872(x-0.500704t)^3 + 0.0311401(x-0.500704t)^4 \\ & -0.0000198611(x-0.500704t)^5 - 0.00220256(x-0.500704t)^6 \\ & +1.72259E^{-6}(x-0.500704t)^7 + 0.000143274(x-0.500704t)^8 \\ & -1.33158E^{-7}(x-0.500704t)^9 - 8.86016E^{-6}(x-0.500704t)^{10}. \end{aligned}$$

Figure 1 below shows the comparison of the DTM approximate solution of order 10 and the exact solution in (3.3). It is clear from figure 1, the DTM approximation and the exact solutions are in good agreement. Also figure 2 below shows the exact solution, approximate solution of $u(x,t)$ for the values of $x = -2, -1, 1, 2$ and $t = 0.2, 0.4, 0.6, 0.8, 1$.

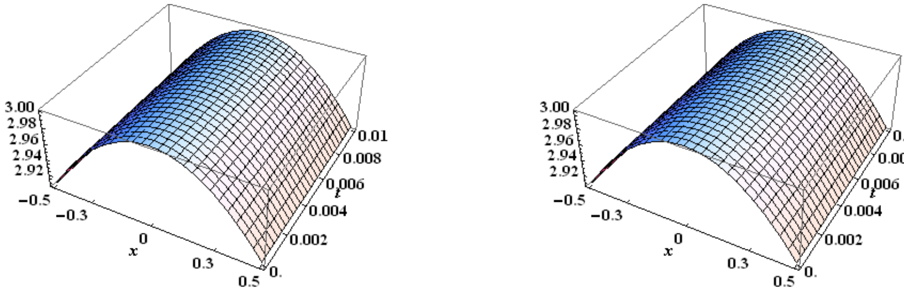

FIGURE 1. The exact, approximate solutions for Example 1

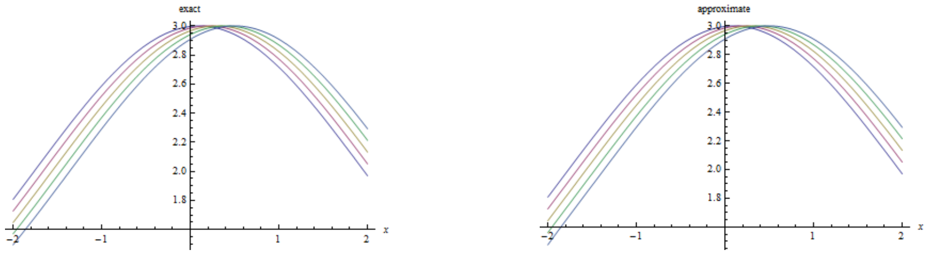

FIGURE 2. The exact and approximate solutions for Example 1 when $-2 < x < 2$ and $0 < t < 1$.

Example 2.

Consider the nonlinear GIKdV in the form ($p = 1, \epsilon = 0.5, \gamma = 1, \nu = 1$):

$$u_t + 0.5\, uu_x + u_{xxx} - u_{xxt} = 0, \tag{3.11}$$

subject to the initial condition

$$u(x, 0) = 3\, sech^2\left(\frac{x}{2\sqrt{3}}\right), \tag{3.12}$$

where the exact solution is

$$u(x, t) = 3\, sech^2\left(\frac{x - 0.5t}{2\sqrt{3}}\right). \tag{3.13}$$

Now using the wave variable $\xi = x - ct$, equations (3.11–3.12) are converted to an ODE and after integrating with respect to ξ we get

$$-cu(\xi) + 0.25u^2(\xi) + (1 - c)u''(\xi) = 0, \tag{3.14}$$

subject to the initial condition

$$u(0) = 3. \tag{3.15}$$

Applying the differential transform to Equations (3.14–3.15) and make a use of Table 1, we obtain the following recursive formula

$$U(k+2) = \frac{cU(k)}{(1 - c)(k + 1)(k + 2)} - \frac{1}{4(1 - c)(k + 1)(k + 2)} \sum_{i=0}^{k} U(i)U(k-i) \tag{3.16}$$

and

$$U(0) = u(0) = 3, \quad U(1) = u'(0) = \beta, \tag{3.17}$$

where β is a constant to be determined. Using $U(0)$, $U(1)$ we coded (3.16) in Mathematica, and obtain the following results:

$$U(2) = \frac{3c^2}{2(c - 1)},\ U(3) = \frac{c\beta}{3(c - 1)}. \tag{3.18}$$

Continuing in this manner, the first 9-iterations can be identified eventually by using mathematica software. Hence, the approximate solution can be expressed as

$$u_{appr}(x, t) \;=\; \sum_{i=0}^{8} U(i)(x - ct)^i. \tag{3.19}$$

Now, using the initial condition (3.12), and by the aid of Mathematica software, the constants c and β are

$$c = 0.49947263613698734, \qquad \beta = 0.00063702031777750952.$$

Substituting the values of c and β in Equation (3.19), then the approximate solution is:

$$\begin{aligned} u_{appr}(x,t) &= 3 + 0.00063702(x - 0.499473t) - 0.250615(x - 0.499473t)^2 \\ &\quad -0.0000708423(x - 0.499473t)^3 + 0.0139353(x - 0.499473t)^4 \\ &\quad +5.02521E^{-6}(x - 0.499473t)^5 - 0.000659002(x - 0.499473t)^6 \\ &\quad -2.91269E^{-7}(x - 0.499473t)^7 + 0.0000286476(x - 0.499473t)^8 \\ &\quad +1.50481E^{-8}(x - 0.499473t)^9. \end{aligned}$$

Figure 3 shows the comparison of the DTM approximate solution of order 9 and the exact solution in (3.13). It is clear from figure 2, the DTM approximation and the exact solutions are in good agreement.

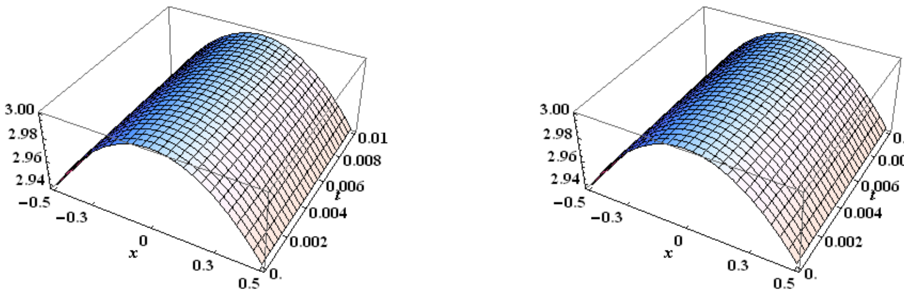

FIGURE 3. The exact, approximate solutions for Example 2

Also figure 4 below shows the exact solution, approximate solution of $u(x,t)$ for the values of $x = -2, -1, 1, 2$ and $t = 0.2, 0.4, 0.6, 0.8, 1$.

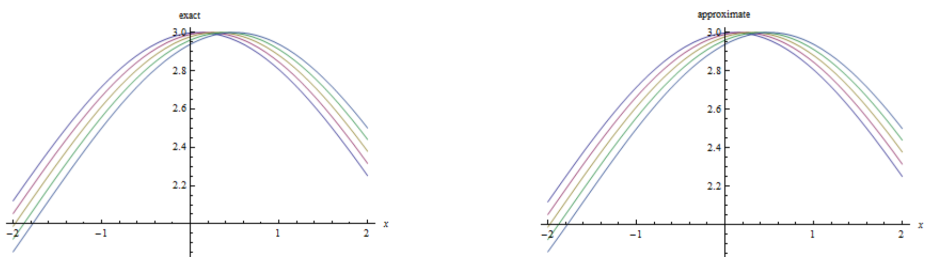

FIGURE 4. The exact and approximate solutions for Example 2 when $-2 < x < 2$ and $0 < t < 1$.

4. APPLICATIONS OF THE ADM

In this section, we apply the ADM to the same previous examples that been considered by the DTM.

4.1. SOLVING GKDV AND GIKDV USING ADM

Now we present two applications of the ADM.

Example 1. Consider the nonlinear GKdV in the form ($p = 1, \epsilon = 0.5, \gamma = 1$):

$$u_t + 0.5\, uu_x + u_{xxx} = 0, \tag{4.1}$$

subject to the initial condition

$$u(x, 0) = 3\, sech^2 \left(\frac{x}{2\sqrt{2}} \right), \tag{4.2}$$

where the exact solution is

$$u(x, t) = 3\, sech^2 \left(\frac{x - 0.5t}{2\sqrt{2}} \right). \tag{4.3}$$

Applying the ADM, Equation (4.1) becomes

$$L_t \left(u(x, t) \right) = -u_{xxx} - 0.5uu_x, \tag{4.4}$$

where L_t is defined by $L_t = \frac{\partial}{\partial t}$. Now the inverse operator L_t^{-1} is identified by

$$L_t^{-1}(.) = \int_0^t (.)\, dz. \tag{4.5}$$

Applying L_t^{-1} to both sides of (4.4) and using the initial condition we obtain

$$u(x, t) - u(x, 0) = -L_t^{-1} u_{xxx} - 0.5 L_t^{-1} uu_x. \tag{4.6}$$

Then

$$u(x, t) = 3\, sech^2 \left(\frac{x}{2\sqrt{2}} \right) - L_t^{-1} u_{xxx} - 0.5 L_t^{-1} uu_x. \tag{4.7}$$

Substituting

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t),$$

and the nonlinear term by

$$0.5uu_x = 0.5 \sum_{n=0}^{\infty} A_n,$$

into Equation (4.7) gives

$$\sum_{n=0}^{\infty} u_n(x, t) = 3\, sech^2 \left(\frac{x}{2\sqrt{2}} \right) - L_t^{-1} \left(\sum_{n=0}^{\infty} (u_n(x, t))_{xxx} \right) - 0.5 L_t^{-1} \left(\sum_{n=0}^{\infty} A_n \right). \tag{4.8}$$

This gives the recursive relation

$$\begin{aligned} u_0(x, t) &= 3\, sech^2 \left(\tfrac{x}{2\sqrt{2}} \right) \\ u_{k+1}(x, t) &= -L_t^{-1} \left((u_k)_{xxx} \right) - L_t^{-1}(A_k),\ k \geq 0. \end{aligned} \tag{4.9}$$

The first two components are given by

$$\begin{aligned} u_0(x,t) &= 3\,sech^2\left(\frac{x}{2\sqrt{2}}\right) \\ u_1(x,t) &= -L_t^{-1}(A_0) = -L_t^{-1}\left(-\frac{9sech^4\left(\frac{x}{2\sqrt{2}}\right)tanh\left(\frac{x}{2\sqrt{2}}\right)}{\sqrt{2}}\right), \end{aligned} \tag{4.10}$$

where additional terms can be easily computed. The Adomian polynomials A_n for this form of nonlinearity are given by

$$\begin{aligned} A_0 &= \frac{u_0(u_0)_x}{2} = -\frac{9sech^4\left(\frac{x}{2\sqrt{2}}\right)tanh\left(\frac{x}{2\sqrt{2}}\right)}{\sqrt{2}} \\ A_1 &= \frac{u_1(u_0)_x}{2} + \frac{u_0(u_1)_x}{2} \\ &= sech^4\left(\frac{x}{2\sqrt{2}}\right)\left(1.125sech^4\left(\frac{x}{2\sqrt{2}}\right) - 54csch^4\left(\frac{x}{\sqrt{2}}\right)\sinh^6\left(\frac{x}{2\sqrt{2}}\right) - 4.5\tanh^4\left(\frac{x}{2\sqrt{2}}\right)\right)t \\ A_2 &= \frac{u_2(u_0)_x}{2} + \frac{u_1(u_1)_x}{2} + \frac{u_0(u_2)_x}{2}. \end{aligned}$$

Combining the results obtained above, the approximate solution is given by

$$\begin{aligned} u_{appr}(x,t) &= 3\,sech^2\left(\frac{x}{2\sqrt{2}}\right) + \left(16.9706 + 16.9706\cosh\left(\frac{x}{\sqrt{2}}\right)\right)csch^5\left(\frac{x}{\sqrt{2}}\right)\sinh^6\left(\frac{x}{2\sqrt{2}}\right)t \\ &+ \left(-0.093750sech^8\left(\frac{x}{2\sqrt{2}}\right) + csch^8\left(\frac{x}{\sqrt{2}}\right)\left(sinh^{10}\left(\frac{x}{2\sqrt{2}}\right) + 72sinh^{12}\left(\frac{x}{2\sqrt{2}}\right) + 48sinh^{14}\left(\frac{x}{2\sqrt{2}}\right)\right)\right)t^2 \\ &+ \ldots \end{aligned}$$

Figure 5 below shows the comparison of the ADM approximate solution of order 3 and the exact solution in (4.3). It is clear from figure 5, the ADM approximation and the exact solution are in excellent agreement. Also figure 6 below shows the exact solution, approximate solution of $u(x,t)$ for the values of $x = -2, -1, 1, 2$ and $t = 0.2, 0.4, 0.6, 0.8, 1$.

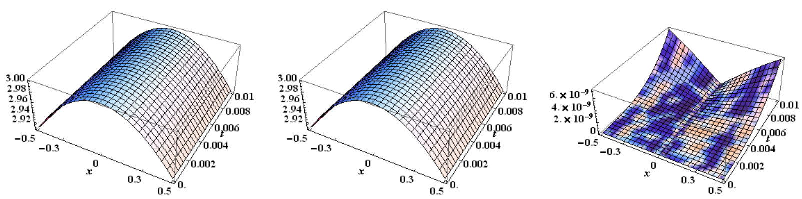

FIGURE 5. The exact, approximate solutions and absolute error, respectively for Example 1

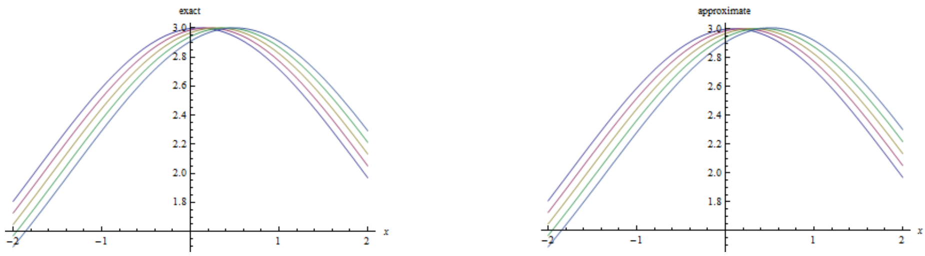

FIGURE 6. The exact and approximate solutions for Example 1 when $-2 < x < 2$ and $0 < t < 1$.

Example 2. Consider the nonlinear GIKdV in the form ($p = 1, \epsilon = 0.5, \gamma = 1, \nu = 1$):

$$u_t + 0.5\, uu_x + u_{xxx} - u_{xxt} = 0, \tag{4.11}$$

subject to the initial condition

$$u(x, 0) = 3\, sech^2\left(\frac{x}{2\sqrt{3}}\right), \tag{4.12}$$

where the exact solution is

$$u(x, t) = 3\, sech^2\left(\frac{x - 0.5t}{2\sqrt{3}}\right). \tag{4.13}$$

Applying the ADM, Equation (4.11) becomes

$$L_t\left(u(x, t)\right) = u_{xxt} - 0.5uu_x - u_{xxx}, \tag{4.14}$$

where L_t is defined by $L_t = \frac{\partial}{\partial t}$. Applying L_t^{-1} to both sides of (4.14) and using the initial condition, we obtain

$$u(x, t) - u(x, 0) = L_t^{-1}u_{xxt} - 0.5L_t^{-1}uu_x - L_t^{-1}u_{xxx}. \tag{4.15}$$

Then

$$u(x, t) = 3\, sech^2\left(\frac{x}{2\sqrt{3}}\right) + L_t^{-1}u_{xxt} - 0.5L_t^{-1}uu_x - L_t^{-1}u_{xxx}. \tag{4.16}$$

Substituting

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t),$$

and the nonlinear term by

$$0.5uu_x = 0.5\sum_{n=0}^{\infty} A_n,$$

into Equation (4.16) gives

$$\sum_{n=0}^{\infty} u_n(x, t) = 3\, sech^2\left(\frac{x}{2\sqrt{3}}\right) + L_t^{-1}\left(\sum_{n=0}^{\infty}(u_n)_{xxt}\right) - L_t^{-1}\left(\sum_{n=0}^{\infty}(u_n)_{xxx}\right) - 0.5L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{4.17}$$

This gives the recursive relation

$$u_0(x,t) = 3\,sech^2\left(\frac{x}{2\sqrt{3}}\right)$$
$$u_{k+1}(x,t) = L_t^{-1}\left((u_k)_{xxt}\right) - L_t^{-1}\left((u_k)_{xxx}\right) - L_t^{-1}(A_k),\ k \geq 0. \tag{4.18}$$

Thus, the first two components are given by

$$u_0(x,t) = 3\,sech^2\left(\frac{x}{2\sqrt{3}}\right)$$
$$u_1(x,t) = -L_t^{-1}(A_0) = -L_t^{-1}\left(3\sqrt{3}sech^4\left(\frac{x}{2\sqrt{3}}\right)\tanh\left(\frac{x}{2\sqrt{3}}\right)\right), \tag{4.19}$$

where additional terms can be easily computed. The Adomian polynomials A_n for this form of nonlinearity are given by

$$A_0 = \frac{u_0(u_0)_x}{2} = -3\sqrt{3}sech^4\left(\frac{x}{2\sqrt{3}}\right)\tanh\left(\frac{x}{2\sqrt{3}}\right). \tag{4.20}$$

Combining the results obtained above, the approximate solution is given by

$$\begin{aligned} u_{appr}(x,t) \quad = \quad & 3\,sech^2\left(\frac{x}{2\sqrt{3}}\right) + 3sec\left(\frac{x}{2}\right)^2\left(\frac{2\sin(2x)}{(1+\cos(x))^2} - \frac{2\sin(x)}{(1+\cos(x))^2}\right)t \\ & + \left(-0.11sech^8\left(\frac{x}{2\sqrt{3}}\right) - 143.99csch^8\left(\frac{x}{\sqrt{3}}\right)sinh^{10}\left(\frac{x}{2\sqrt{3}}\right)\right)t^2 \\ & + \left(\left(110.22\cosh\left(\frac{x}{\sqrt{3}}\right) + 1.78\cosh\left(\frac{2x}{\sqrt{3}}\right)\right)csch^8\left(\frac{x}{\sqrt{3}}\right)sinh^{10}\left(\frac{x}{2\sqrt{3}}\right)\right)t^2 \\ & + \ldots \end{aligned}$$

Figure 7 below shows the comparison of the ADM approximate solution of order 3 and the exact solution in (4.13). It is clear from figure 7, the ADM approximation and the exact solution are in excellent agreement.

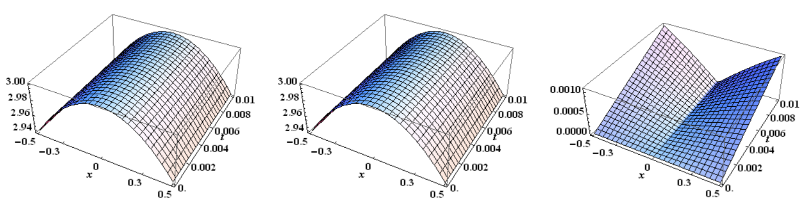

FIGURE 7. The exact, approximate solutions and absolute error, respectively for Example 2

5. CONCLUSION

In this paper, we successfully apply the DTM and ADM were used to find approximate solutions to the GKdV and GIKdV. The present methods reduces the computational difficulties of the other traditional methods and all the calculations can be made simple manipulations. Two examples were tested by applying the DTM and ADM. The results have shown remarkable performance. Therefore, these methods can be applied to many nonlinear integral and differential equations without linearization, discretization or perturbation.

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Conflict of Interests

The author declare that there is no conflict of interests regarding the publication of this paper.

References

[1] G. Adomian, Solving frontier problems of physics: the decomposition method. *Kluwer Acad. Publ.*, (1994).
[2] G. Adomian, A review of the decomposition method in applied mathematics. *J. Math. Anal. Appl.*,Volume 135, No.10, (1988) 501-544.
[3] G. Adomian, Nonlinear stochastic operator equations. *Academic Press, New York*, (1986).
[4] G. Adomian, A new approach to nonlinear partial differential equations. *J. Math. Anal. Appl.*, Vol. 102, (1984), 420-434.
[5] G. Adomian, Explicit solutions of nonlinear partial differential equations. *J. Math. Anal. Appl.*, 88, (1997) 117-126.
[6] A. Ali, A. Soliman and K. Raslan, Soliton solution for nonlinear partial differential equations by cosine-function method. *Physics Letters A*, 368, (2007) 299-304.
[7] A. S. V. Kanth, K. Aruna, differential transform method for solving linear and non-linear systems of partial differential equations. *Physics Letters A*, Volume 372, (2008), 6896-6898.
[8] Z. M. Odibat, Differential transform method for solving Volterra integral Equations with separable kernels. *Math. Comput. Model*, Volume 48, Issues 78, (2008), 1144-1149.
[9] G. E. Pukhov, Differential transforms and circuit theory. *Int. J. Circ. Theor. App*, **10**, (1982), 265.
[10] G. E. Pukhov, Differential transforms of functions and equations, in Russian. *Naukova Dumka, Kiev*, (1980).
[11] G. E. Pukhov, Differential transformations and mathematical modeling of physical processes, in Russian. *Naukova Dumka, Kiev*, (1986).
[12] M. Rawashdeh, Improved Approximate Solutions for Nonlinear Evolutions Equations in Mathematical Physics Using the RDTM. *Journal of Applied Mathematics and Bioinformatics*, Volume 3, No. 2, (2013) 1-14.
[13] M. Rawashdeh, Using the Reduced Differential Transform Method to Solve Nonlinear PDEs Arises in Biology and Physics. *World Applied Sciences Journal*, Volume 23, No. 8, (2013), 1037-1043.
[14] M. Rawashdeh, N. Obeidat, On Finding Exact and Approximate Solutions to Some PDEs Using the Reduced Differential Transform Method. *Appl. Math. Inf. Sci.*,(2014). Vol. 8, No. 5, 1-6.
[15] M. Rawashdeh, N. Obeidat, Applying the Reduced Differential Transform Method to Solve the Telegraph and Cahn-Hilliard Equations. *Thai Journal of Mathematics*, (To appear)(2014).
[16] Abdul Majid. Wazwaz, Partial Differential Equations and Solitary Waves Theory. *Springer-Verlag, Heidelberg*, (2009).
[17] G. B. Whitham, Linear and Nonlinear Waves. *John Wiley, New York*, (1976).

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