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BALL CONVERGENCE FOR VARIANTS OF JARRATT'S METHOD

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Abstract We present a local convergence analysis for some fourth-order multipoint variants of Jarratt's method which are optimal. The local convergence of these methods is shown under hypotheses up to the fourth derivative of the function although only the first derivative appears in these methods [18]. In the present paper we show convergence using only the first derivative. This way the applicability of these methods is expanded. Computable radii of convergence and error bounds are provided using Lipschitz constants. Numerical examples where our result apply but earlier ones do not apply to solve equations are also given in this study.

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1. Introduction

In this paper the problem of approximating a locally unique solution x^* of equation

$$F(x) = 0 \tag{1.1}$$

is analyzed. Here where F is a Frécher differentiable operator defined on a subset D of a Banach space X with values in a Banach space Y. Newton-like methods are widely used for finding solution of (1.1), these methods are usually studied based on: semi-local and local convergence. The semi-local convergence method is based on the information around an initial point, to give conditions ensuring the convergence of the iterative procedure; while the local one is, based on the information around a solution, to find estimates of the radii of convergence balls [3, 4, 22–24, 26].

Third order methods such as Euler's, Halley's, super Halley's, Chebyshev's [1]-[31] require the evaluation of the second derivative F'' at each step, which in general is very

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expensive. That is why many authors have used higher order multi-point methods [1]-[31]. In this paper, we study the local convergence of some third order variants of Jarratt's method defined for each $n = 0, 1, 2, \cdots$ by

$$\begin{aligned} y_n &= x_n - \frac{2}{3}F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= x_n - F'(x_n)^{-1}F(x_n)A_n, \\ A_n &= I + \frac{21}{8}F'(x_n)^{-1}F'(y_n) - \frac{9}{2}(F'(x_n)^{-1}F'(y_n))^2 + \frac{15}{8}(F'(x_n)^{-1}F'(y_n))^3, \end{aligned} \tag{1.2}$$

where x_0 is an initial point. This method is studied in [?] in the special case when $X = Y = \mathbb{R}^m$. The local convergence of these methods was shown using hypotheses up to the fourth Fréchet–derivative of operator F although only the first derivative appears in these methods. These hypotheses limit the applicability of method (1.2). As a motivational let us define function f on $D = [-\frac{1}{2}, \frac{5}{2}]$ by

$$f(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Choose $x^* = 1$. We have that

$$\begin{aligned} f'(x) &= 3x^2 \ln x^2 + 5x^4 - 4x^3 + 2x^2, \; f'(1) = 3, \\ f''(x) &= 6x \ln x^2 + 20x^3 - 12x^2 + 10x \\ f'''(x) &= 6 \ln x^2 + 60x^2 - 24x + 22. \end{aligned}$$

Then, function f''' is unbounded on D. In the present paper we only use hypotheses on the first Fréchet derivative. This way we expand the applicability of these methods. Notice that, in-particular there is a plethora of iterative methods for approximating solutions of nonlinear equations defined on X [1]–[31]. These results show that if the initial point x_0 is sufficiently close to the solution x^*, then the sequence $\{x_n\}$ converges to x^*. But how close to the solution x^* the initial guess x_0 should be? These local results give no information on the radius of the convergence ball for the corresponding method. We address this question for method (1.2) in Section 2. The same technique can be used to other methods. Our results are also presented in affine invariant form.

The rest of the paper is organized as follows. The local convergence of methods (1.2) are given in Section 2, whereas the numerical examples are given in the concluding Section 3.

2. Local Convergence Analysis

We present the local convergence analysis of method (1.2) in this section. Let $L_0 > 0$, $L > 0$ and $M \geq 1$ be given parameters. It is convenient for the local convergence analysis of method (1.2) that follows to define some functions and parameters. Define function g_1 on the interval $[0, \frac{1}{L_0})$ by:

$$g_1(t) = \frac{1}{2(1 - L_0 t)}(Lt + \frac{2M}{3}),$$

and parameters r_1 and r_A by

$$r_1 = \frac{2(1 - M/3)}{2L_0 + L}, \;\; r_A = \frac{2}{2L_0 + L}.$$

Suppose that

$$M < 3 \tag{2.1}$$

Then, we have by (2.1) that $0 < r_1 < r_A$ and for each $t \in [0, r_1)$, $0 \le g_1(t) < 1$. Moreover, define functions g_2 and h_2 on the interval $[0, \frac{1}{L_0})$ by

$$\begin{aligned} g_2(t) &= \frac{1}{2(1-L_0t)}[Lt + \frac{21Mg_1(t)}{4} \\ &+ \frac{9M^2g_1^2(t)t}{1-L_0t} + \frac{15M^3g_1^3(t)t^2}{(1-L_0t)^2}] \end{aligned}$$

and

$$h_2(t) = g_2(t) - 1.$$

Suppose

$$M < \sqrt{\frac{8}{7}}. \tag{2.2}$$

Then, we have by (2.2) that $h_2(0) = \frac{7M^2}{8} - 1 < 0$ and $h_2(t) \to +\infty$ as $t \to \frac{1}{L_0}^-$. Denote by r_2 the smallest zero of function h_2 in the interval $(0, \frac{1}{L_0})$.. Set

$$r = \min\{r_1, r_2\} \tag{2.3}$$

Notice that hypothesis (2.2) implies (2.1). Then, we have

$$0 < r < r_A \tag{2.4}$$

and for each $t \in [0, r)$

$$0 \le g_1(t) < 1 \tag{2.5}$$

and

$$0 \le g_2(t) < 1. \tag{2.6}$$

Denote by $U(v, \rho)$, $\bar{U}(v, \rho)$, respectively for the open and closed balls in X with center $v \in X$ and of radius $\rho > 0$. Next, we present the local convergence of method (1.2) using the preceding notation.

Theorem 2.1. *Let $F : D \subset X \to Y$ be a Fréchet–differentiable operator. Suppose that there exist $x^* \in D$, $L_0 > 0$, $L > 0$, $M \ge 1$ such that for each $x, y \in D$, (2.2),*

$$F(x^*) = 0, \ F'(x^*)^{-1} \in L(Y, X), \tag{2.7}$$

$$\|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \le L_0\|x - x^*\|, \tag{2.8}$$

$$\|F'(x^*)^{-1}(F'(x) - F'(y))\| \le L\|x - y\|, \tag{2.9}$$

$$\|F'(x^*)^{-1}F'(x)\| \le M, \tag{2.10}$$

and

$$\bar{U}(x^*, r) \subseteq D, \tag{2.11}$$

hold, where the radius r is given by (2.3). Then, the sequence $\{x_n\}$ generated for $x_0 \in U(x^*, r) - \{x^*\}$ by method (1.2) is well defined, remains in $U(x^*, r)$ for each $n = 0, 1, 2, ...$ and converges to x^*. Moreover the following estimates hold

$$\|y_n - x^*\| \le g_1(\|x_n - x^*\|)\|x_n - x^*\| < \|x_n - x^*\| < r \tag{2.12}$$

and

$$\|x_{n+1} - x^*\| \leq g_2(\|x_n - x^*\|)\|x_n - x^*\| < \|x_n - x^*\|, \tag{2.13}$$

where "g" functions are defined above Theorem 2.1. Furthermore, if there exists $T \in [r, 2/L_0)$ *such that* $\bar{U}(x^*, T) \subseteq D$*, then the limit point* x^* *is the only solution of equation* $F(x) = 0$ *in* $\bar{U}(x^*, T)$*.*

Proof. We shall show estimates (2.12) and (2.13) using mathematical induction. By hypothesis $x_0 \in U(x^*, r) - \{x^*\}$, (2.3) and (2.8), we have that

$$\|F'(x^*)^{-1}(F'(x_0) - F'(x^*))\| \leq L_0\|x_0 - x^*\| < L_0 r < 1. \tag{2.14}$$

It follows from (2.14) and the Banach Lemma on invertible functions [3, 4, 25, 28], that $F'(x_0)^{-1} \in L(Y, X)$ and

$$\|F'(x_0)^{-1}F'(x^*)\| \leq \frac{1}{1 - L_0\|x_0 - x^*\|} < \frac{1}{1 - L_0 r}. \tag{2.15}$$

Hence y_0 and x_1 are well deined. We can write by (2.7) that

$$F(x_0) = F(x_0) - F(x^*) = \int_0^1 F'(x^* + \theta(x_0 - x^*))(x_0 - x^*)d\theta. \tag{2.16}$$

Notice that $\|x^* + \theta(x_0 - x^*) - x^*\| = \theta\|x_0 - x^*\| \leq \|x_0 - x^*\| < r$. That is $x^* + \theta(x_0 - x^*) \in U(x^*, r)$. Then, using (2.10) and (2.16), we get that

$$\|F'(x^*)^{-1}F(x_0)\| = \|\int_0^1 F'(x^*)^{-1}F'(x^* + \theta(x_0 - x^*))(x_0 - x^*)d\theta\| \leq M\|x_0 - x^*\|. \tag{2.17}$$

Using (2.3), (2.5), (2.9), (2.11) and (2.17) we obtain in turn that

$$\begin{aligned} \|y_0 - x^*\| &\leq \|x_0 - x^* - F'(x_0)^{-1}F(x_0)\| \\ &\quad + \frac{1}{3}\|F'(x_0)^{-1}F(x_0)\| \\ &\leq \|F'(x_0)^{-1}F'(x^*)\|\|\int_0^1 F'(x^*)^{-1}(F'(x^* + \theta(x_0 - x^*)) - F'(x_0))(x_0 - x^*)d\theta\| \\ &\quad + \frac{1}{3}\|F'(x_0)^{-1}F'(x^*)\|\|F'(x^*)\| \\ &\leq \frac{L\|x_0 - x^*\|}{2(1 - L_0\|x_0 - x^*\|)} + \frac{M\|x_0 - x^*\|}{3(1 - L_0\|x_0 - x^*\|)} \\ &= g_1(\|x_0 - x^*\|)\|x_0 - x^*\| < \|x_0 - x^*\| < r, \end{aligned} \tag{2.18}$$

which shows (2.12) for $n = 0$ and $y_0 \in U(x^*, r)$. Then, using the second substep of method (1.2), (2.6), (2.15) and (2.17) we get in turn that

$$\begin{aligned}
\|x_1 - x^*\| &\leq \|x_0 - x^* - F'(x_0)^{-1}F(x_0)\| \\
&+\frac{21}{8}\|F'(x_0)^{-1}F'(x^*)\|\|F'(x^*)^{-1}F(y_0)\| \\
&+\frac{9}{2}(\|F'(x_0)^{-1}F'(x^*)\|\|(F'(x^*)^{-1}F(y_0)\|)^2 \\
&+\frac{15}{8}(\|F'(x_0)^{-1}F'(x^*)\|\|F'(x^*)^{-1}F'(y_0)\|)^3 \\
&\leq \frac{L\|x_0 - x^*\|^2}{2(1 - L_0\|x_0 - x^*\|)} + \frac{21M\|y_0 - x^*\|}{8(1 - L_0\|x_0 - x^*\|)} \\
&+\frac{9M^2\|y_0 - x^*\|^2}{2(1 - L_0\|x_0 - x^*\|)^2} + \frac{15M^3\|y_0 - x^*\|^3}{8(1 - L_0\|x_0 - x^*\|)^3} \\
&\leq g_2(\|x_0 - x^*\|)\|x_0 - x^*\| < \|x_0 - x^*\| < r, \qquad (2.19)
\end{aligned}$$

which shows (2.13) for $n = 0$ and $x_1 \in U(x^*, r)$. By simply replacing x_0, y_0, x_1 by x_k, y_k, x_{k+1} in the preceding estimates we arrive at estimates (2.12) and (2.13). Using the estimate $\|x_{k+1} - x^*\| < \|x_k - x^*\| < r$, we deduce that $x_{k+1} \in U(x^*, r)$ and $\lim_{k\to\infty} x_k = x^*$. To show the uniqueness part, let $B = \int_0^1 F'(y^* + \theta(x^* - y^*)d\theta$ for some $y^* \in \bar{U}(x^*, T)$ with $F(y^*) = 0$. Using (2.8) we get that

$$\begin{aligned}
\|F'(x^*)^{-1}(B - F'(x^*))\| &\leq \int_0^1 L_0\|y^* + \theta(x^* - y^*) - x^*\|d\theta \\
&\leq L_0\int_0^1 (1-\theta)\|x^* - y^*\|d\theta \leq \frac{L_0}{2}T < 1. \qquad (2.20)
\end{aligned}$$

It follows from (2.20) and the Banach Lemma on invertible functions that B is invertible. Finally, from the identity $0 = F(x^*) - F(y^*) = B(x^* - y^*)$, we deduce that $x^* = y^*$.

□

Remark 2.2. 1. In view of (2.8) and the estimate

$$\begin{aligned}
\|F'(x^*)^{-1}F'(x)\| &= \|F'(x^*)^{-1}(F'(x) - F'(x^*)) + I\| \\
&\leq 1 + \|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \leq 1 + L_0\|x - x^*\|
\end{aligned}$$

condition (2.10) can be dropped and M can be replaced by

$$M(t) = 1 + L_0 t, \text{ for each } t \in [0, \frac{1}{L_0})$$

or simply by $M = 2$, since $t \in [0, \frac{1}{L_0})$.

2. The results obtained here can be used for operators F satisfying autonomous differential equations [3] of the form

$$F'(x) = P(F(x))$$

where P is a continuous operator. Then, since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing x^*. For example, let $F(x) = e^x - 1$. Then, we can choose: $P(x) = x + 1$.

3. The radius r_A was shown by us to be the convergence radius of Newton's method [3], [4]

$$x_{n+1} = x_n - F'(x_n)^{-1}F(x_n) \text{ for each } n = 0, 1, 2, \cdots \tag{2.21}$$

under the conditions (2.8) and (2.9). It follows from the definition of r that the convergence radius r of the method (1.2) cannot be larger than the convergence radius r_A of the second order Newton's method. As already noted in [3, 4] r_A is at least as large as the convergence ball given by Rheinboldt [23]

$$r_R = \frac{2}{3L}. \tag{2.22}$$

In particular, for $L_0 < L$ we have that

$$r_R < r$$

and

$$\frac{r_R}{r_A} \to \frac{1}{3} \ as \ \frac{L_0}{L} \to 0.$$

That is our convergence ball r_A is at most three times larger than Rheinboldt's. The same value for r_R was given by Traub [24].

4. It is worth noticing that method (1.2) is not changing when we use the conditions of Theorem 2.1 instead of the stronger conditions used in [18]. Moreover, we can compute the computational order of convergence (COC) defined by

$$\xi = \ln\left(\frac{\|x_{n+1} - x^*\|}{\|x_n - x^*\|}\right) / \ln\left(\frac{\|x_n - x^*\|}{\|x_{n-1} - x^*\|}\right)$$

or the approximate computational order of convergence

$$\xi_1 = \ln\left(\frac{\|x_{n+1} - x_n\|}{\|x_n - x_{n-1}\|}\right) / \ln\left(\frac{\|x_n - x_{n-1}\|}{\|x_{n-1} - x_{n-2}\|}\right).$$

This way we obtain in practice the order of convergence in a way that avoids the bounds involving estimates using estimates higher than the first Fréchet derivative of operator F.

3. Numerical Examples

We present numerical examples in this section.

Example 3.1. Returning back to the motivational example at the introduction of this study, we have $L_0 = L = 146.6629073$, $M = 1 + L_0 t$, $t \in (0, \frac{\sqrt{8/7}-1}{L_0})$. The parameters for methods (1.2) are

$$r_A = 0.0045,\ r_1 = 0.0045,\ r_R = 0.0045,\ r_2 = 0.0004.$$

Example 3.2. Let $X = Y = \mathbb{R}^3, D = \bar{U}(0,1), x^* = (0,0,0)^T$. Define function F on D for $w = (x,y,z)^T$ by

$$F(w) = (e^x - 1, \frac{e-1}{2}y^2 + y, z)^T.$$

Then, the Fréchet-derivative is given by

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Notice that using the $(\mathcal{C})$ conditions, we get $L_0 = e-1, L = e, M = 1 + L_0 t$, $t \in (0, \frac{\sqrt{8/7}-1}{L_0})$. The parameters for method (1.2) are

$$r_A = 0.3249,\ r_1 = 0.3206,\ r_R = 0.2453,\ r_2 = 0.0208.$$

Example 3.3. Let $X = Y = C[0,1]$, the space of continuous functions defined on $[0,1]$ and be equipped with the max norm. Let $D = \overline{U}(0,1)$ and $B(x) = F''(x)$ for each $x \in D$. Define function F on D by

$$F(\varphi)(x) = \varphi(x) - 5\int_0^1 x\theta\varphi(\theta)^3 d\theta. \tag{3.1}$$

We have that

$$F'(\varphi(\xi))(x) = \xi(x) - 15\int_0^1 x\theta\varphi(\theta)^2\xi(\theta)d\theta, \text{ for each } \xi \in D.$$

Then, we get that $x^* = 0$, $L_0 = 7.5$, $L = N = 15$, $M = 1 + L_0 t$, $t \in (0, \frac{\sqrt{8/7}-1}{L_0})$. The parameters for method (1.2) are

$$r_A = 0.0667,\ r_1 = 0.0065,\ r_R = 0.0444,\ r_2 = 0.0042.$$

Conflict of Interests

The author declare that there is no conflict of interests regarding the publication of this paper.

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