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FIXED POINT RESULTS IN PARTIALLY ORDERED COMPLEX VALUED METRIC SPACES FOR RATIONAL TYPE EXPRESSIONS

Binayak S. Choudhury[1]**, Nikhilesh Metiya**[2,*] **and Pulak Konar**[3]
[1] *Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah - 711103, West Bengal, India*
E-mails: binayak12@yahoo.co.in
[2] *Department of Mathematics, Sovarani Memorial College, Jagatballavpur, Howrah-711408, West Bengal, India*
E-mails: metiya.nikhilesh@gmail.com
[3] *Department of Mathematics, NITMAS, South 24 Pargana, West Bengal, 743368, India*
E-mails: pulakkonar@gmail.com

**Corresponding author.*

Abstract In this paper, we establish some fixed point results for mappings involving rational type contractions in the framework of complex valued metric spaces endowed with a partial order. The proved results generalize and extend some known results in the literature. Two illustrative examples are given.

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1. Introduction

Metric fixed point theory is widely recognized to have been originated in the work of S. Banach in 1922 [1], where he proved the famous contraction mapping principle. Banach's contraction mapping principle has very few parallels in modern science, in terms of the various influences it has had in the developments of different branches of mathematics and of physical science in general. Over the years metric fixed point theory has developed in different directions. A comprehensive account of this development provided in the handbook entitled by Kirk and Sims [2].

Also there are large efforts for generalizing metric spaces by changing the form and interpretation of the metric function. Ghaler [3] introduced 2-metric spaces where a real number is assigned to any three points of the space. Probabilistic metric spaces were

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introduced by Schweiter et al [4, 5] in which any pair of points is assigned to a suitable distribution function making possible a probabilistic sense of distance. Fuzzy metric spaces were introduced in more than one ways by various means of fuzzification as, for example in [6] by assigning any pair of points to a suitable fuzzy set and spelling out the triangular inequality by using a t-norm. Another example is in the work of Kaleva et al [7] where any pair of points is assigned to a fuzzy number. Cone metric spaces were introduced by Huang et al [8] where the metric is allowed to take up values in real Banach spaces. G-metric space [9] is another generalization in which every triplet of points is assigned to a non-negative real number but in a different way than in 2-metric spaces. There are also other extension of the metric which are not mentioned above. It can be seen that in recent times efforts of extending the concept of metric space has continued in a rapid manner. Simultaneously, metric fixed point theory has been extended rapidly in these spaces over the recent years.

Complex valued metric space is a recently introduced generalization of metric space where the metric function takes values from the field of complex numbers, thus opening the scope of the concepts from complex analysis for incorporation in the metric space structure. The space was originally introduced by Azam et al [10]. Fixed point theory has been studied in this space in a suitable number of papers, some of which we mention in ([11] - [15]).

Dass and Gupta [16] generalized the Banach's contraction mapping principle by using a contractive condition of rational type. Fixed point theorems for contractive type conditions satisfying rational inequalities in metric spaces have been developed in a number of works ([17] - [23]).

Fixed point theory in partially ordered metric spaces is of relatively recent origin. An early result in this direction is due to Turinici [24] in which fixed point problems were studied in partially ordered uniform spaces. Later, this branch of fixed point theory has developed through a number of works some of which are in ([25] - [35]).

The purpose of this paper is to study fixed points of a class of mappings satisfying a rational expression in the frame-work of a partially ordered complex valued metric space.

2. Preliminaries

Let $\mathcal{C}$ be the set of complex numbers and $z_1,\ z_2 \in \mathcal{C}$. Define a partial order $\precsim$ on $\mathcal{C}$ as follows:

$$z_1 \precsim z_2 \ \text{ if and only if } \ Re(z_1) \leq Re(z_2) \ \text{ and } \ Im(z_1) \leq Im(z_2).$$

It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

$$(i) \quad Re(z_1) = Re(z_2), \ \ Im(z_1) < Im(z_2),$$

$$(ii) \quad Re(z_1) < Re(z_2), \ \ Im(z_1) = Im(z_2),$$

$$(iii) \quad Re(z_1) < Re(z_2), \ \ Im(z_1) < Im(z_2),$$

$$(iv) \quad Re(z_1) = Re(z_2), \ \ Im(z_1) = Im(z_2).$$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied.

Note that

$$z_1 \precsim z_2, \ \ z_2 \prec z_3 \implies z_1 \prec z_3.$$

Definition 2.1 ([10])**.** Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathcal{C}$ satisfies:

$$\begin{array}{ll} (i) & 0 \precsim d(x,\ y),\ \text{for all } x,\ y \in X \text{ and } d(x,y) = 0 \text{ if and only if } x = y, \\ (ii) & d(x,\ y) = d(y,\ x),\ \text{for all } x,\ y \in X, \\ (iii) & d(x,\ y) \precsim d(x,\ z) + d(z,\ y),\ \text{for all } x,y,z \in X. \end{array}$$

Then d is called a complex valued metric on X and $(X,\ d)$ is called a complex valued metric space.

Definition 2.2. Let $(X,\ d)$ be a complex valued metric space, $\{x_n\}$ be a sequence in X and $x \in X$.
(i) If for every $c \in \mathcal{C}$ with $0 \prec c$ there is $n_0 \in \mathbf{N}$ such that for all $n > n_0$, $d(x_n,\ x) \prec c$, then $\{x_n\}$ said to be convergent and $\{x_n\}$ converges to x. We denote this by $\lim_{n \longrightarrow \infty} x_n = x$, or $x_n \longrightarrow x$ as $n \longrightarrow \infty$.
(ii) If for every $c \in \mathcal{C}$ with $0 \prec c$ there is $n_0 \in \mathbf{N}$ such that for all $n,\ m > n_0$, $d(x_n,\ x_m) \prec c$, then $\{x_n\}$ is said to be a Cauchy sequence.
(iii) If every Cauchy sequence in X is convergent, then $(X,\ d)$ is called a complete complex valued metric space.

Lemma 2.3 ([10])**.** Let $(X,\ d)$ be a complex valued metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ converges to x if and only if $d(x_n,\ x) \longrightarrow 0$ as $n \longrightarrow \infty$.

Note 2.4. We can also replace the limit in lemma 2.3 by the equivalent limiting condition $|d(x_n,\ x)| \longrightarrow 0$ as $n \longrightarrow \infty$.

Lemma 2.5. *Let $(X,\ d)$ be a complex valued metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n,\ x_m) \longrightarrow 0$ as $n, m \longrightarrow \infty$.*

Note 2.6. We can also replace the limit in lemma 2.5 by the equivalent limiting condition $|d(x_n,\ x_m)| \longrightarrow 0$ as $n,\ m \longrightarrow \infty$.

Definition 2.7. Let $(X,\ d)$ be a complex valued metric space, $T : X \longrightarrow X$ and $x \in X$. Then the function T is continuous at x if for any sequence $\{x_n\}$ in X,

$$x_n \longrightarrow x \quad \Longrightarrow \quad Tx_n \longrightarrow Tx.$$

Definition 2.8. Let $(X,\ \preceq)$ be a partially ordered set and $T : X \longrightarrow X$. The mapping T is said to be nondecreasing if for all $x_1,\ x_2 \in X$, $x_1 \preceq x_2$ implies $Tx_1 \preceq Tx_2$ and nonincreasing if for all $x_1,\ x_2 \in X$, $x_1 \preceq x_2$ implies $Tx_1 \succeq Tx_2$.

3. Main Results

Theorem 3.1. *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there exists a complex valued metric d on X such that $(X,\ d)$ is complete complex valued metric space. Let $T : X \longrightarrow X$ be a continuous and nondecreasing mapping. Suppose there exist non-negative real numbers α, β and γ with $\alpha + \beta + \gamma < 1$ such that, for all $x,\ y \in X$ with $x \preceq y$,*

$$d(Tx,\ Ty) \precsim \alpha\ d(x,\ y) + \beta\ \frac{d(y,\ Ty)\ [1 + d(x,\ Tx)]}{1 + d(x,\ y)} + \gamma\ \frac{d(y,\ Tx)\ [1 + d(x,\ Ty)]}{1 + d(x,\ y)}. \quad (3.1)$$

If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point.

Proof. If $x_0 = Tx_0$, then we have the result. Suppose that $x_0 \prec Tx_0$. Then we construct a sequence $\{x_n\}$ in X such that

$$x_{n+1} = Tx_n, \quad \text{for every} \quad n \geq 0. \tag{3.2}$$

Since T is a nondecreasing mapping, we obtain by induction that

$$x_0 \prec Tx_0 = x_1 \preceq Tx_1 = x_2 \preceq \ldots \preceq Tx_{n-1} = x_n \preceq Tx_n = x_{n+1} \preceq \ldots \tag{3.3}$$

If there exists some $N \geq 1$ such that $x_{N+1} = x_N$, then from (3.2), $x_{N+1} = Tx_N = x_N$; that is, x_N is a fixed point of T and the proof is finished. So, we suppose that $x_{n+1} \neq x_n$, for all $n \geq 1$. Since $x_n \prec x_{n+1}$, for all $n \geq 1$, applying (3.1) we have

$$\begin{aligned} d(x_{n+1},\ x_{n+2}) &= d(Tx_n,\ Tx_{n+1}) \\ &\precsim \alpha\ d(x_n,\ x_{n+1}) + \beta\ \frac{d(x_{n+1},\ x_{n+2})[1 + d(x_n,\ x_{n+1})]}{1 + d(x_n,\ x_{n+1})} \\ &\quad + \gamma \frac{d(x_{n+1},\ x_{n+1})[1 + d(x_n, x_{n+2})]}{1 + d(x_n,\ x_{n+1})}; \end{aligned}$$

that is,

$$d(x_{n+1},\ x_{n+2}) \precsim \frac{\alpha}{1-\beta}\ d(x_n,\ x_{n+1}). \tag{3.4}$$

Put $\dfrac{\alpha}{1-\beta} = h\ (< 1)$. Then it follows that

$$d(x_{n+1},\ x_{n+2}) \precsim h\ d(x_n,\ x_{n+1}) \precsim h^2\ d(x_{n-1},\ x_n) \precsim \ldots \precsim h^{n+1}\ d(x_0,\ x_1). \tag{3.5}$$

For any $m > n$,

$$\begin{aligned} d(x_m,\ x_n) &\precsim d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2}) + \ldots + d(x_{m-1},\ x_m) \\ &\precsim [h^n + h^{n+1} + h^{n+2} + \ \ldots\ + h^{m-1}]\ d(x_0,\ x_1) \\ &\precsim \frac{h^n}{1-h}\ d(x_0,\ x_1) \longrightarrow 0 \ \text{ as } \ n \longrightarrow \infty, \end{aligned}$$

which implies that $\{x_n\}$ is a Cauchy sequence. From the completeness of X, there exists a $z \in X$ such that

$$x_n \longrightarrow z \quad \text{as} \quad n \longrightarrow \infty. \tag{3.6}$$

The continuity of T implies that $Tz = \lim_{n \longrightarrow \infty} Tx_n = \lim_{n \longrightarrow \infty} x_{n+1} = z$; that is, z is a fixed point of T. ∎

In our next theorem we relax the continuity assumption of the mapping T in Theorem 3.1 by imposing the following order condition of the complex valued metric space X:

If $\{x_n\}$ is a non-decreasing sequence in X such that $x_n \longrightarrow x$, then $x_n \preceq x$, for all $n \in \mathbf{N}$.

Theorem 3.2. *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there exists a complex valued metric d on X such that $(X,\ d)$ is complete complex valued metric space. Assume that if $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \longrightarrow x$, then $x_n \preceq x$, for all $n \in \mathbf{N}$. Let $T : X \longrightarrow X$ be a nondecreasing mapping. Suppose that (3.1) holds for all $x,\ y \in X$ with $x \preceq y$. If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point.*

Proof. We take the same sequence $\{x_n\}$ as in the proof of theorem 3.1. Similar to the proof of Theorem 3.1, we prove that $\{x_n\}$ is a nondecreasing sequence such that $x_n \longrightarrow z \in X$. Then $x_n \preceq z$, for all $n \in \mathbf{N}$. Applying (3.1), we have

$$\begin{aligned} d(x_{n+1},\ Tz) &= d(Tx_n,\ Tz) \\ &\precsim \alpha\ d(x_n,\ z) + \beta\ \frac{d(z,Tz)[1+d(x_n,x_{n+1})]}{1+d(x_n,z)} + \gamma\ \frac{d(z,x_{n+1})[1+d(x_n,Tz)]}{1+d(x_n,z)}. \end{aligned}$$

Taking the limit as $n \longrightarrow \infty$ in the above inequality and using (3.6), we have

$$d(z,\ Tz) \precsim \beta\ d(z,\ Tz).$$

Since $\beta < 1$, it is a contradiction unless $d(z,\ Tz) = 0$; that is, $Tz = z$, that is, z is a fixed point of T. ∎

Now, we shall prove the uniqueness of the fixed point.

Theorem 3.3. *In addition to the hypotheses of theorem 3.1 (or theorem 3.2), suppose that for every $x,\ y \in X$, there exists $u \in X$ such that $u \preceq x$ and $u \preceq y$. Then T has a unique fixed point.*

Proof. It follows from the theorem 3.1 (or theorem 3.2) that the set of fixed points of T is non-empty. We shall show that if x^* and y^* are two fixed points of T; that is, if $x^* = Tx^*$ and $y^* = Ty^*$, then $x^* = y^*$.

By the assumption, there exists $u_0 \in X$ such that $u_0 \preceq x^*$ and $u_0 \preceq y^*$. Then, similarly as in the proof of theorem 3.1, we define the sequence $\{u_n\}$ such that

$$u_{n+1} = Tu_n = T^{n+1}u_0, \quad n = 0, 1, 2, ... \tag{3.7}$$

Monotonicity of T implies that

$$T^n u_0 = u_n \preceq x^* = T^n x^* \ \text{ and } \ T^n u_0 = u_n \preceq y^* = T^n y^*.$$

If there exists a positive integer m such that $x^* = u_m$, then $x^* = Tx^* = Tu_n = u_{n+1}$, for all $n \geq m$. Then $u_n \longrightarrow x^*$ as $n \longrightarrow \infty$. Now we suppose that $x^* \neq u_n$, for all $n \geq 0$. So $u_n \prec x^*$, for all $n \geq 0$. Applying (3.1), we have

$$\begin{aligned} d(u_{n+1}, x^*) &= d(Tu_n, Tx^*) \\ &\precsim \alpha\ d(u_n, x^*) + \beta\ \frac{d(x^*, Tx^*)[1+d(u_n,u_{n+1})]}{1+d(u_n,x^*)} + \gamma\ \frac{d(x^*,u_{n+1})[1+d(u_n,Tx^*)]}{1+d(u_n,x^*)} \\ &= \alpha\ d(u_n, x^*) + \beta\ \frac{d(x^*, x^*)[1+d(u_n,u_{n+1})]}{1+d(u_n,x^*)} + \gamma\ \frac{d(x^*,u_{n+1})[1+d(u_n,x^*)]}{1+d(u_n,x^*)} \\ &= \alpha\ d(u_n, x^*) + \gamma\ d(x^*, u_{n+1}), \end{aligned}$$

which implies that

$$d(u_{n+1},\ x^*) \precsim \frac{\alpha}{1-\gamma}\ d(u_n,\ x^*).$$

Put $\frac{\alpha}{1-\gamma} = h\ (< 1)$. Then it follows that

$$d(u_{n+1},\ x^*) \precsim h\ d(u_n,\ x^*) \precsim h^2\ d(u_{n-1},\ x^*) \precsim ... \precsim h^{n+1}\ d(u_0,\ x^*) \longrightarrow 0 \ \text{ as } \ n \longrightarrow \infty.$$

Hence

$$d(u_n,\ x^*) \longrightarrow 0 \ \text{ as } \ n \longrightarrow \infty; \text{ that is, } u_n \longrightarrow x^* \ \text{ as } \ n \longrightarrow \infty. \tag{3.8}$$

Using a similar argument, we can prove that

$$u_n \longrightarrow y^* \quad \text{as} \quad n \longrightarrow \infty. \tag{3.9}$$

Finally, the uniqueness of the limit implies that $x^* = y^*$. Hence T has a unique fixed point. ∎

Example 3.4. Let $X = \{0,\ \frac{1}{2},\ 2\}$. Partial order $' \preceq '$ is defined as $x \preceq y$ iff $x \geq y$. Let the complex valued metric d be given as

$$d(x,\ y) = |x - y|\ \sqrt{2}\ e^{i\frac{\pi}{4}} = |x - y|(1 + i),\ \text{for } x,\ y \in X.$$

Let $T : X \longrightarrow X$ be defined as follows:

$$T(0) = 0,\quad T(\frac{1}{2}) = 0,\quad T(2) = \frac{1}{2}.$$

Let $\alpha = \frac{1}{2}, \beta = \gamma = \frac{1}{8}$.

Here all the conditions of Theorems 3.1 and 3.1 are satisfied. Additionally the conditions of Theorem 3.3 are also satisfied and it is seen that 0 is the unique fixed point of T.

Example 3.5. Let $X = [1.5,\ 2]$ with usual prtial order $' \leq '$. Let us consider the complex valued metric d as defined in example 3.4.

Let $T : X \longrightarrow X$ be defined as follows:

$$Tx = \begin{cases} 1.81, & \text{if} \quad 1.5 \leq x < 1.75, \\ x + \frac{1}{x} - \frac{1}{2}, & \text{if} \quad 1.75 \leq x \leq 2. \end{cases}$$

Let $\alpha = 0.005, \beta = .01, \gamma = 0.98$.

Here all the conditions of Theorems 3.2 and 3.3 are satisfied and $x = 2$ is the unique fixed point of T.

Remark 3.6. In the above example the function T is not continuous and hence it is not applicable to theorem 3.1.

4. Conclusion

In our result we use products and quotients of the metric values which is permissible in the structure of complex numbers. But this is not always the case with other generalizations of metric spaces as, for example, in cone metric, where the metric is real Banach space valued, we cannot use products and quotients of metric values.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] S. Banach, Sur les oprations dans les ensembles abstraits et leurs applications aux quations intgrales, Fund Math., 3(1922) 133-181.

[2] W. A. Kirk and B. Sims, Handbook of metric fixed point theory, 2001, XIII, 703 p.

[3] S. Gahler, Uber die unifromisieberkeit 2-metrischer Raume, Math. Nachr., 28 (1965) 235 - 244.

[4] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. Math., 10 (1960) 314-334.

[5] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, Dover Pub. Incorporated, 2011.

[6] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets System, 64 (1994) 395-399.

[7] O. Kaleva and S. Seikkala, On fuzzy metric spaces, Fuzzy Sets Systems, 12 (1984) 215-229.

[8] L. G. Huang and X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl., 332 (2007) 1468-1476.

[9] Z. Mustafa and B. Sims, Some remarks concerning D-metric spaces, Proc. Int. Conf. on Fixed Point Theory Appl. Valencia Spain July, (2003) 189-198.

[10] A. Azam, B. Fisher and M. Khan, Common fixed point theorems in complex valued metric spaces, Numer. Funct. Anal. Optim., 32 (2011) 243-253.

[11] A. Azam, J. Ahmad and P. Kumam, Common fixed point theorems for multi-valued mappings in complex-valued metric spaces, Journal of Inequalities and Applications, 2013 (2013): 578.

[12] S. Bhatt, S. Chaukiyal and R. C. Dimri, A common fixed point theorem for weakly compatible maps in complex-valued metric spaces, Int. J. Math. Sci. Appl., 1 (2011) 1385-1389.

[13] W. Sintunavarat and P. Kumam, Generalized common fixed point theorems in complex valued metric spacers and applications, J. Inequal Appl, 2012 (2012): 84.

[14] K. Sitthikul and S. Saejung, Some fixed point theorems in complex valued metric space, Fixed Point Theory Appl., 2012 (2012) : 189.

[15] R. K. Verma and H. K. Phatak, Common fixed point theorems using property (E.A) in complex valued metric spaces, Thai. J. Math., 11 (2013) 347 - 355.

[16] B. K. Dass and S. Gupta, An extension of Banach contraction principle through rational expressions, Inidan J. Pure Appl. Math., 6 (1975) 1455-1458.

[17] S. Chandok and J. K. Kim, Fixed point theorem in ordered metric spaces for generalized contractions mappings satisfying rational type expressions, J. Nonlinear Functional Anal. Appl., 17 (2012) 301-306.

[18] S. Chandok, M. S. Khan and K. P. R. Rao, Some coupled common fixed point theorems for a pair of mappings satisfying a contractive condition of rational type, J. Nonlinear Anal. Appl., 2013 (2013) 1-6.

[19] J. Harjani, B. López, and K. Sadarangani, A fixed point theorem for mappings satisfying a contractive condition of rational type on a partially ordered metric space, Abstract Appl. Anal., 2010, Article ID 190701.

[20] D. S. Jaggi, Some unique fixed point theorems, Indian J Pure Appl Math., 8 (1977) 223-230.

[21] D. S. Jaggi and B. K. Das, An extension of Banach's fixed point theorem through rational expression, Bull. Cal. Math. Soc., 72 (1980) 261-264.

[22] N. V. Luong and N. X. Thuan, Fixed point theorem for generalized weak contractions satisfying rational expressions in ordered metric spaces, Fixed Point Theory Appl., 46 (2011) 1-10.
[23] B. G. Pachpatte, Common fixed point theorems for mappings satisfying rational inequalities, Indian J. pure appl. Math., 10 (1979) 1362-1368.
[24] M. Turinici, Abstract comparison principles and multivariable Gronwall-Bellman inequalities, J. Math. Anal. Appl., 117 (1986) 100-127.
[25] B. S. Choudhury and A. Kundu, (ψ, α, β) - weak contractions in partially ordered metric spaces, Appl. Math. Lett., 25 (2012) 6-10.
[26] B. S. Choudhury and P. Maity, Coupled fixed point results in generalized metric spaces, Math. Comput. Modelling, 54 (2011) 73-79.
[27] B. S. Choudhury and N. Metiya, Multivalued and singlevalued fixed point results in partially ordered metric spaces, Arab J. Math. Sci., 17 (2011) 135-151.
[28] B. S. Choudhury and N. Metiya, Fixed point and common fixed point results in ordered cone metric spaces, An. Şt. Univ. Ovidius Constanţa, 20 (2012) 55-72.
[29] B. S. Choudhury, K. Das and P. Das, Coupled coincidence point results for compatible mappings in partially ordered fuzzy metric spaces, Fuzzy Sets and Systems, 222 (2013) 84-97.
[30] B. S. Choudhury, N. Metiya and M. Postolache, A generalized weak contraction principle with applications to coupled coincidence point problems, Fixed Point Theory Appl., 2013 (2013) : 152.
[31] J. Harjani and K. Sadarangani, Fixed point theorems for weakly contractive mappings in partially ordered sets, Nonlinear Anal., 71 (2009) 3403-3410.
[32] J. Harjani and K. Sadarangani, Generalized contractions in partially ordered metric spaces and applications to ordinary differential equations, Nonlinear Anal., 72 (2010) 1188-1197.
[33] P. Kaushik, S. Kumar and P. Kumam, Coupled coincidence point theorems for $\alpha - \psi$ - contractive type mappings in partially ordered metric spaces, Fixed Point Theory Appl., 2013 (2013) : 325.
[34] J. J. Nieto and R. López, Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Order, 22 (2005) 223-239.
[35] A. C. M. Ran and M. C. B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc. Amer. Math. Soc., 132 (2004) 1435-1443.]

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Theoretical and Computational Science Center (TaCS)
Science Laboratory Building, Faculty of Science
King Mongkuts University of Technology Thonburi (KMUTT)
126 Pracha Uthit Road, Bang Mod, Thung Khru, Bangkok, Thailand 10140
Website: http://tacs.kmutt.ac.th/
Email: tacs@kmutt.ac.th

