EXACT ANALYSIS OF UNSTEADY CONVECTIVE DIFFUSION FOR BLOOD FLOW WITH INTERPHASE MASS TRANSFER IN MAGNETIC FIELD

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Abstract This paper deals with a mathematical model for exact analysis of miscible dispersion of solute with interphase mass transfer in a blood(couple stress fluid) flow bounded by porous beds under the influence of magnetic field. The three coefficients, namely, exchange coefficient, convection coefficient, and dispersion coefficient are evaluated asymptotically at large-time using generalized dispersion model. The dispersion equation is used to calculate the mean concentration distribution of a solute, bounded by the porous layer, and is expressed as a function of dimensionless axial distance and time. It is computed for different values of Hartmann number ($M$), Couple Stress Parameter($a$), reaction rate Parameter($\beta$) and Porous Parameter ($\sigma$).

MSC: 76S05, 76W05,76D05

Keywords: magnetic field, couple stress fluid, porous medium, generalized dispersion model.

1. INTRODUCTION

In many biomedical problems, the interphase mass transfer plays an important role because many physiological situation involve interphase mass transfer. Therefore, it is necessary to develop a technique for handling such problems, which involve interphase mass transport. Several authors have studied various characteristics of dispersion were mainly concerned with Taylors dispersion [23], which is valid for large time. Physiological fluid flow problems have been mainly concerned with transient phenomena where Taylors model is not valid. However, Sankarasubramanian and Gill[18] have developed an analytical method to analyze transient dispersion of non-uniform initial distribution called
generalized dispersion in laminar flow in a tube with a first order chemical reaction at the tube wall. This method can be applied to physiological problems, where a first order chemical reaction occurs at the tube wall. One such situation is transport of oxygen and nutrients to tissue cells and removal of metabolic waste products from tissue cells. Inter-phase mass transfer also takes place in pulmonary capillaries, where the carbon dioxide is removed from the blood and oxygen is taken up by the blood.

Rudraiah et al. [17] studied the dispersion in a stokes couple-stress fluid flow by using the generalized model of Gill and Sankarasubramanian [8]. Considering solute reaction at the channel walls in their all time analysis of dispersion, reaction at the walls is of practical interest and in the simplest case, a first order chemical reaction at the walls is considered by them, in carrying out an exact analysis of unsteady convection in couple stress fluid flows.

Gill [7] developed a local theory of Taylor diffusion in fully developed laminar tube flow with a periodic condition at the inlet of the tube. Gill [8] obtained an exact solution to the unsteady convective diffusion equation for miscible displacement in fully developed laminar flow in tubes by defining dispersion coefficient to be functions of time. Shivakumar et al. [20] have obtained a closed-form solution for unsteady convective diffusion in a fluid-saturated sparsely packed porous medium using the generalized dispersion model of [8]. In all these investigations, it is assumed that the solute does not chemically react in the liquid in which it is dispersed. Gupta [9], following Taylor [23], studied the effects of homogeneous and heterogeneous reaction on the dispersion of a solute in the laminar flow of Newtonian fluid between two parallel plates.

Shukla et al. [19], Soundalgekar [21], Meena Priya [10], Dulal Pal [3] and Dutta et al. [4] studied dispersion in non-Newtonian fluids by considering only homogeneous first-order chemical reaction in the bulk of the fluid. Chandra and Agarwal [2] considered dispersion in simple microfluid flows taking only homogeneous reaction into consideration. Suvadip Paul [22] examine the influence of angularity on the transport process under the combined effects of reversible and irreversible wall reactions, when the flow is driven by a pressure gradient comprising of steady and periodic components. Francesco Gentile [5] studied the transport formulation proposed in [6] was further developed to account for the time dependency of the problem. Prathap Kumar et al. [11] also investigated the effect of homogeneous and heterogeneous reactions on the solute dispersion in composite porous medium. Recently, Ramana Rao [13], Ramana [14, 15] and Ramana et al. [16] studied combined the effect of non-Newtonian rheology and irreversible boundary reaction on dispersion in a Herschel-Bulkley fluid through a conduit, (pipe/channel) by using the generalized dispersion model proposed by [8]. Pauling [12] first reported that the erythrocytes orient with their disk plane parallel to the magnetic field.

This paper deals with the effect of couple stress and magnetic field on the unsteady convective diffusion, with interphase mass transfer by using the generalized dispersion model of Sankarasubramanian and Gill [18]. Convection coefficient $K_1$ and Dispersion coefficient $K_2$ are influenced by the couple stress parameter arising due to suspension in the fluid, magnetic field and porous parameter. The exchange coefficient $K_0$ arises mainly due to the interphase mass transfer, and it is independent of the solvent fluid velocity. The interphase mass transfer also influence the convection and dispersion coefficients.
2. **Mathematical Formulation**

We have considered a steady laminar and fully developed flow (unidirectional) in a channel bounded by porous layers and separated by a distance $2h$. A schematic diagram of the physical configuration and the description of the initial slug input of concentration are shown in figure 1. A uniform magnetic field $B_0$ is applied in the $y$-direction to the flow of blood. We make the following assumption for electromagnetic interactions, i) the induce magnetic field and the electric field produced by the motion of blood are negligible (since blood has low magnetic Reynolds number). ii) no external electric field is applied. Flow region is divided into two sub-region such as Fluid film region and Porous tissue region. The governing equation of the motion for flow in vector form is given by

**Region 1: Fluid Film Region**

Conservation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0$$  \hspace{1cm} (2.1)

Conservation of momentum

$$\rho \left( \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right) = -\nabla p + \mu \nabla^2 \vec{q} - \lambda \nabla^4 \vec{q} + J \times B$$  \hspace{1cm} (2.2)

**Region 2: Porous Tissue Region**

Conservation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0$$  \hspace{1cm} (2.3)

Conservation of momentum

$$\rho \left( \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right) = -\nabla p + \mu \nabla^2 \vec{q} - \frac{\mu}{k} (1 + \beta_1) \vec{q}$$  \hspace{1cm} (2.4)

Maxwell’s equations are

$$\nabla \cdot B = 0, \quad \nabla \times B = \mu_0 J$$

$$\nabla \times E = -\frac{\partial B_0}{\partial t}$$

Ohm’s law

$$J = \sigma_0(E + \vec{q} \times B_0)$$
Conservation of species
\[ \frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla)C = D \nabla^2 C \]  \hspace{1cm} (2.5)

In cartesian form, using the above equation (2.1)-(2.4) becomes

Region 1: Fluid Film Region
\[ 0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \lambda \frac{\partial^4 u}{\partial y^4} - B_0^2 \sigma_0 u, \]  \hspace{1cm} (2.6)
\[ 0 = -\frac{\partial p}{\partial y} \]  \hspace{1cm} (2.7)

Region 2: Porous Tissue Region
\[ 0 = -\frac{\partial p}{\partial x} - \frac{\mu}{k}(1 + \beta_1)u_p, \]  \hspace{1cm} (2.8)
\[ 0 = -\frac{\partial p}{\partial y} \]  \hspace{1cm} (2.9)

where, \( \vec{q} = iu + jv \), \( u \) is the \( x \) components of velocity, \( p \) is the pressure, \( \rho \) is the density of the fluid, \( \mu \) is the viscosity of the fluid, \( \lambda \) is the couple stress parameter, \( k \) is the permeability of the porous medium, \( E \) the electric field, \( \sigma_0 \) is the electrical conductivity, \( J \) the current density and \( u_p \) is the darcy velocity. It may be noted that, (2.8) is the modified darcy equation. where, \( \beta_1 \) is the couple stress parameter.

We consider the dispersion of reactive solute in the fully developed flow through a parallel plate channel bounded by porous beds. Introduced a slug of concentration \( C = C_0 \psi_1(x)Y_1(y) \). The mass balance equation (2.5) concerning the solute concentration \( C \) undergoes heterogeneous chemical reaction such as
\[ \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \]  \hspace{1cm} (2.10)

where, \( D \) is the molecular diffusivity.

The boundary conditions on velocity are
\[ \frac{\partial u}{\partial y} = -\alpha \sqrt{k} (u - u_p) \text{ at } y = h, \]  \hspace{1cm} (2.11)
\[ \frac{\partial u}{\partial y} = \alpha \sqrt{k} (u - u_p) \text{ at } y = -h, \]  \hspace{1cm} (2.12)
\[ \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = \pm h. \]  \hspace{1cm} (2.13)

where, \( \alpha \) is the slip parameter. Eqs. (2.11) and (2.12) is Beavers and Joseph [1] slip condition at the lower and upper permeable surfaces and equation (2.13) specifies the vanishing of the couple stress.

Initial and Boundary conditions on concentration

The initial distribution assumed to be in a variable separable form is given by
\[ C(0, x, y) = C_0 \psi_1(x)Y_1(y), \]  \hspace{1cm} (2.14)

The heterogeneous reaction conditions are:
\[ \begin{align*}
-D \frac{\partial C}{\partial y} &= K_s C \quad \text{at } y = h \text{ and } \\
D \frac{\partial C}{\partial y} &= K_s C \quad \text{at } y = -h
\end{align*} \]  \hspace{1cm} (2.15)
where, \( K_s \) is the reaction rate constant catalysed by the walls and \( C_0 \) is a reference concentration.

As the amount of solute in the system is finite,

\[
C(t, \infty, y) = \frac{\partial C}{\partial x}(t, \infty, y) = 0 \quad (2.16)
\]

where, \( C_0 \) is reference concentration.

Now, we introduce the non-dimensional quantities,

\[
U = \frac{u}{\bar{u}}; \quad \eta = \frac{y}{h}; \quad \theta = \frac{C}{C_0}; \quad X = \frac{Dx}{h^2 \bar{u}}; \quad X_s = \frac{Dx_s}{h^2 \bar{u}}; \quad \tau = \frac{Dt}{h^2}; \quad Pe = \frac{h\bar{u}}{D};
\]

Equation (2.6) and (2.10) in non-dimensional form as

\[
\frac{\partial^4 U}{\partial \eta^4} - a^2 \frac{\partial^2 U}{\partial \eta^2} + a^2 M^2 U = Pa^2 \quad (2.17)
\]

and

\[
\frac{\partial \theta}{\partial \tau} + U^* \frac{\partial \theta}{\partial X_1} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial X_1^2} + \frac{\partial^2 \theta}{\partial \eta^2} \quad (2.18)
\]

where, \( P = \frac{h^2}{\mu} \frac{\partial p}{\partial x}, l = \sqrt{\frac{\lambda}{\mu}} \) and \( a = \frac{h}{t} \) is the couple stress parameter, \( M^2 = \frac{B^2_0 \sigma h^2}{\mu} \) is the square of the Hartmann number, \( Pe = \frac{h \bar{u}}{D} \) is the peclet number, \( U^* = \frac{U - \bar{U}}{U} \) is normalized axial component of velocity as \( x_1 = x - \bar{u}t \) which is dimensionless form is \( X_1 = X - \tau \) where \( X_1 = \frac{X - \tau h^2}{D} \).

The initial and boundary conditions of (2.11) to (2.16) in dimensionless form

\[
\frac{\partial U}{\partial \eta} = -\alpha \sigma (U - U_p) \text{ at } \eta = 1 \quad (2.19)
\]

\[
\frac{\partial U}{\partial \eta} = \alpha \sigma (U - U_p) \text{ at } \eta = -1 \quad (2.20)
\]

\[
\frac{\partial^2 U}{\partial \eta^2} = 0 \text{ at } y = \pm 1 \quad (2.21)
\]

where, \( \sigma = \frac{h}{\sqrt{k}} \) is the porous parameter.

\[
\theta(0, X, \eta) = \psi(X)Y(\eta), \quad (2.22)
\]

\[
\begin{align*}
\frac{\partial \theta}{\partial \eta} &= -\beta \theta \quad \text{ at } \eta = 1, \\
\frac{\partial \theta}{\partial \eta} &= \beta \theta \quad \text{ at } \eta = -1
\end{align*} \quad (2.23)
\]

\[
\theta(\tau, \infty, \eta) = \frac{\partial \theta}{\partial X}(\tau, \infty, \eta) = 0 \quad (2.24)
\]
3. Method of Solution

Velocity distribution
The solution of Equation (2.17) can be written as:

\[ U(\eta) = C_1e^{m_1\eta} + C_2e^{-m_1\eta} + C_3e^{m_3\eta} + C_4e^{-m_3\eta} + \frac{P}{M^2} \]  

(3.1)

where, \( C_1, C_2, C_3 \) and \( C_4 \) are constants. Applying the boundary conditions (2.19)-(2.21) in (3.1), we obtain the velocity of blood as

\[ U(\eta) = 2C_1Coshm_1\eta + 2C_3Coshm_3\eta + \frac{P}{M^2} \]  

(3.2)

The normalized axial components of velocity obtained from (3.1) is

\[ U^* = U - \bar{U} = 2A_1 \left[ C_1Coshm_1\eta + C_3Coshm_3\eta - \left( \frac{C_1Sinhm_1}{m_1} + \frac{C_3Sinhm_3}{m_3} \right) \right] \]  

(3.3)

where,

\[ \bar{U} = \frac{1}{2} \int_{-1}^{1} U(\eta)d\eta = \frac{2C_1Sinhm_1}{m_1} + \frac{2C_3Sinhm_3}{m_3} + \frac{P}{M^2} \]  

(3.4)

Generalized Dispersion model
The solution of (2.18) subject to the conditions (2.22)-(2.24), following Gill and Sankara-subramanian [18] is

\[ \theta(\tau, X, \eta) = \sum_{k=0}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial X^k}, \]  

(3.5)

where, \( \theta_m \) is the dimensionless cross sectional average concentration and is given by

\[ \theta_m = \frac{1}{2} \int_{-1}^{1} \theta(\tau, X, \eta)d\eta \]  

(3.6)

Equation (2.18) is multiplied throughout by \( \frac{1}{2} \) and integrated with respect to \( y \) between the limits -1 to 1 and using (3.6) we get,

\[ \frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2} \frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2} \left[ \frac{\partial \theta}{\partial \eta} \right]_{-1}^{1} - \frac{1}{2} \frac{\partial}{\partial X} \int_{-1}^{1} U^* \theta d\eta \]  

(3.7)

Using Eq. (3.5) in (3.7), we get the dispersion model for \( \theta_m \) as

\[ \frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2} \frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2} \left[ \frac{\partial}{\partial \eta} (f_0(\tau, \eta)\theta_m(\tau, X) + f_1(\tau, \eta)\frac{\partial \theta_m}{\partial X}(\tau, X) + \ldots) \right]_{-1}^{1} \]

\[ - \frac{1}{2} \frac{\partial}{\partial X} \int_{-1}^{1} U^*(f_0(\tau, \eta)\theta_m(\tau, X) + f_1(\tau, \eta)\frac{\partial \theta_m}{\partial X}(\tau, X) + \ldots)d\eta \]  

(3.8)
The generalized dispersion model of [7] is defined as

\[
\frac{\partial \theta_m}{\partial \tau} = \sum_{i=0}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial X^i}
\]  

(3.9)

substituting (3.9) in (3.8) we get,

\[
K_0 \theta_m + K_1 \frac{\partial \theta_m}{\partial X} + K_2 \frac{\partial^2 \theta_m}{\partial X^2} = \frac{1}{P_e^2} \frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2} \left[ \frac{\partial}{\partial \eta} (f_0 \theta_m + f_1 \frac{\partial \theta_m}{\partial X} + \ldots) \right]_{-1}^{1} - \frac{1}{2} \frac{\partial}{\partial X} \int_{-1}^{1} U^*(f_0(\tau, \xi) \theta_m(\tau, X) + f_1(\tau, \xi) \frac{\partial \theta_m}{\partial X}(\tau, X) + \ldots) d\eta 
\]

Equating like coefficient of \( \theta_m, \frac{\partial \theta_m}{\partial X}, \frac{\partial^2 \theta_m}{\partial X^2} \ldots \) we get \( K_i \)'s are,

\[
K_i(\tau) = \frac{\delta_{i2}}{P_e^2} + \frac{1}{2} \frac{\partial f_i}{\partial \eta}(\tau, 1) - \frac{1}{2} \int_{-1}^{1} f_{i-1}(\tau, \eta) U^*(\tau, \eta) d\eta 
\]

(3.10)

where, \( f_{-1} = 0 \) and \( \delta_{i2} \) is the Kronecker delta defined by

\[
\delta_{i2} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}
\]

The exchange coefficient \( K_0(\tau) \) accounts for the non-zero solute flux at the channel wall, and negative sign indicates the depletion of solute in the system with time caused by the irreversible reaction, which occurs at the channel wall. The presence of non-zero solute flux at the walls of the channel, also affects the higher order \( K_i \) due to the explicit appearance of \( \frac{\partial f_i}{\partial \eta}(\tau, 1) \) in equation (3.10). Equation (3.9) can be truncated after the term involving \( K_2 \) without causing serious error, because \( K_3, K_4, \) etc. become negligibly small compared to \( K_2 \). The resulting model for the mean concentration is

\[
\frac{\partial \theta_m}{\partial \tau} = K_0(\tau) \theta_m + K_1(\tau) \frac{\partial \theta_m}{\partial X} + K_2(\tau) \frac{\partial^2 \theta_m}{\partial X^2}
\]

(3.11)

To solve the equation (3.11), we need the coefficients \( K_i(\tau) \) in addition to the appropriate initial and boundary conditions. For this, the corresponding function \( f_k \) must be determined. So, substituting (3.5) into (2.18), the following set of differential equations for \( f_k \) are generated.

\[
\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial \eta^2} - U^* f_{k-1} + \frac{1}{P_e^2} f_{k-2} + \sum_{i=0}^{k} K_i f_{k-i}, \quad (k = 0, 1, 2, \ldots)
\]

(3.12)

where, \( f_{-1} = f_{-2} = 0 \).
To evaluate $K_i's$, we need to know the $f_k's$ which are obtained by solving (3.12) for $f_k's$ subject to the boundary conditions,

$$f_k(τ,0) = \text{finite}, \quad (3.13)$$

$$\frac{∂f_k}{∂η}(τ,1) = -βf_k(τ,1), \quad (3.14)$$

$$\frac{∂f_k}{∂η}(τ,0) = 0, \quad (3.15)$$

$$\frac{1}{2} \int_{-1}^{1} f_k(τ,η)dη = δ_{k0}, (k = 0, 1, 2) \quad (3.16)$$

The function $f_0$ and the exchange coefficient $K_0$ are independent of the velocity and can be solved easily. It should be pointed out here, that, a simultaneous solution has to be obtained from these two quantities since $K_0$, which can be obtained from (3.10) as

$$K_0(τ) = \frac{1}{2} \left[ \frac{∂f_0}{∂η} \right]_{-1}^{1} \quad (3.17)$$

Substituting $k = 0$ in equation (3.12) we get the differential equation for $f_0$ as

$$\frac{∂f_0}{∂τ} = \frac{∂^2f_0}{∂η^2} - f_0K_0 \quad (3.18)$$

We derive an initial condition for $f_0$ using (3.6) by taking $τ = 0$ in that equation to get

$$θ_m(0,X) = \frac{1}{2} \int_{-1}^{1} θ(0,X,η)dη \quad (3.19)$$

Substituting $τ = 0$ in (3.5) and setting $f_k(η) = 0 (k = 1, 2, 3)$ gives us the initial condition for $f_0$ as

$$f_0(0,η) = \frac{θ(0,X,η)}{θ_m(0,X)} \quad (3.20)$$

We note that the left hand side of (3.20) is a function of $η$ only and the right hand side is a function of both $X$ and $η$. Thus, clearly the initial concentration distribution must be a separable function of $X$ and $η$. Substituting equation (2.14) and (3.19) into equation (3.20), we get

$$f_0(0,η) = \frac{ψ(η)}{\frac{1}{2} \int_{-1}^{1} ψ(η)dη} \quad (3.21)$$

The solution of the reaction diffusion equation (3.18) with these conditions may be formulated as

$$f_0(τ,η) = g_0(τ,η) \ exp \left[ -τ \int_{0}^{τ} K_0(η)dη \right] \quad (3.22)$$

from which it follows that $g_0(τ,η)$ has to satisfy

$$\frac{∂g_0}{∂τ} = \frac{∂^2g_0}{∂η^2} \quad (3.23)$$
with conditions

\[
f_0(0, \eta) = g_0(0, \eta) = \frac{1}{2} \int_{-1}^{1} \psi(\eta) d\eta,
\]
\[
g_0(\tau, 0) = \text{finite},
\]
\[
\frac{\partial g_0}{\partial \eta}(\tau, 1) = -\beta g_0(\tau, 1).
\]

The solution of (3.23) subject to conditions (3.24)-(3.26) is

\[
g_0(\tau, \eta) = \sum_{n=0}^{\infty} A_n \cos(\mu_n \eta) \exp[-\mu_n^2 \tau]
\]

where, \(\mu_n\)'s are the roots of

\[
\mu_n \tan \mu_n = \beta, \quad n = 0, 1, 2, \ldots
\]

and \(A_n\)'s are given by

\[
A_n = \frac{2}{\left(1 + \frac{\sin 2\mu_n}{2\mu_n}\right)} \int_{-1}^{1} \psi(\eta) \cos \mu_n \eta d\eta
\]

from (3.22), it follows that

\[
f_0(\tau, \eta) = \frac{2g_0(\tau, \eta)}{\int_{-1}^{1} g_0(\tau, \eta) d\eta} = \sum_{n=0}^{9} A_n \exp[-\mu_n^2 \tau] \cos \mu_n \eta
\]

\[
K_0(\tau) = -\sum_{n=0}^{9} \frac{A_n \mu_n \exp[-\mu_n^2 \tau]}{\sum_{n=0}^{9} A_n \exp[-\mu_n^2 \tau]} \sin \mu_n
\]

\(K_0(\tau)\) is independent of velocity distribution.

As \(\tau \to \infty\), we get the asymptotic solution for \(K_0\) from (3.17) in the form

\[
K_0(\infty) = -\mu_0^2
\]

where, \(\mu_0\) is the first root of the equation (3.28). Physically, this represents first order chemical reaction coefficient having obtained \(K_0(\infty)\). We can now get \(K_1(\infty)\), from (3.10) (with \(i = 1\)) knowing \(f_0(\infty, \eta)\) and \(f_1(\infty, \eta)\). Likewise, \(K_2(\infty), K_3(\infty), \ldots\) require the knowledge of \(K_0, K_1, f_0, f_1\) and \(f_2\). Equation (3.30) in the limit \(\tau \to \infty\) reduces to

\[
f_0(\infty, \eta) = \frac{\mu_0}{\sin \mu_0} \cos(\mu_0 \eta)
\]
Then we find \( f_1, K_1, f_2, \) and \( K_2 \). For asymptotically long times, i.e., \( \tau \to \infty \), equation (3.10) and (3.12) give us \( K_i \)'s and \( f_k \)'s as

\[
K_i(\infty) = \frac{\delta_{i2}}{P_2^2} - \beta f_i(\infty, 1) - \int_{-1}^{1} U^* f_{i-1}(\infty, \eta) d\eta, \quad (i = 1, 2, 3) \tag{3.34}
\]

\[
d^2 f_k \frac{d\eta^2}{d\eta} + \mu_0^2 f_k = (U^* + K_1) f_{k-1} - \left( \frac{1}{P_2^2} - K_2 \right) f_{k-2}, \quad (k = 1, 2) \tag{3.35}
\]

The \( f_k \)'s must satisfy the conditions (3.6) and this permits the eigen function expansion in the form of

\[
f_k(\infty, \eta) = \sum_{j=0}^{9} B_{j,k} \cos(\mu_j \eta), \quad k = 1, 2, 3, \ldots \tag{3.36}
\]

Substituting (3.36) in (3.35) and multiplying the resulting equation by \( \cos(\mu_j \eta) \) and integrating with respect to \( \eta \) from -1 to 1,

\[
B_{j,k} \cos(\mu_j \eta) = \frac{1}{\mu_j^2 - \mu_0^2} \left[ \frac{1}{P_2^2} \sum_{j=0}^{\infty} B_{j,k-2} \cos(\mu_j \eta) - U^* \sum_{j=0}^{\infty} B_{j,k-1} \cos(\mu_j \eta) - \sum_{j=0}^{\infty} K_i B_{j,k-i} \cos(\mu_j \eta) \right]
\]

multiplying by \( \cos(\mu_j \eta) \) and integrating with respect to \( \eta \), we get

\[
B_{j,k} = \frac{1}{\mu_j^2 - \mu_0^2} \left[ \frac{1}{P_2^2} B_{j,k-2} - \sum_{i=1}^{k} K_i B_{j,k-i} - \left( 1 + \frac{\sin(2\mu_j)}{2\mu_j} \right)^{-1} \sum_{j=0}^{9} B_{j,k-1} I(j, l) \right] \quad k = (1, 2) \tag{3.37}
\]

where,

\[
I(j, l) = \int_{-1}^{1} U^* \cos(\mu_j \eta) \cos(\mu_l \eta) d\eta = I(l, j) \tag{3.38}
\]

\[
B_{j,-1} = 0, B_{j,0} = 0 \quad \text{for} \quad j = 1 \to 9 \tag{3.39}
\]

The first expansion coefficient \( B_{0,k} \) in equation (3.36) using conditions (3.13)-(3.16) can be expressed in terms of \( B_{j,k} \) \( (j = 1 \to 9) \) as,

\[
f_k = B_{0,k} \cos(\mu_0 \eta) + \sum_{j=1}^{\infty} B_{j,k} \cos(\mu_j \eta) \quad \text{(from equation 3.36)}
\]

By integrating this equation, we get

\[
0 = B_{0,k} \frac{\sin(\mu_0 \eta)}{\mu_0} + \sum_{j=1}^{\infty} B_{j,k} \frac{\sin(\mu_j \eta)}{\mu_j}
\]

(Using the boundary condition \( \int_{-1}^{1} f_k(\tau, \eta) d\eta = \delta_{k0} = 0 \))

\[
B_{0,k} = - \left( \frac{\mu_0}{\sin(\mu_0)} \right) \sum_{j=1}^{9} B_{j,k} \frac{\sin(\mu_j)}{\mu_j}, \quad (k = 1, 2, 3, \ldots) \tag{3.40}
\]
Further, from (3.36) and (3.33) we find that
\[ B_{0,0} = \frac{\mu_0}{\sin \mu_0} \] (3.41)

Substituting \( i = 1 \) in (3.34) and using (3.38), (3.39) and (3.41) in the resulting equation, we get
\[ K_1(\infty) = - \frac{I(0,0)}{\left[ 1 + \frac{\sin 2\mu_0}{2\mu_0} \right]} \] (3.42)

Substituting \( i = 2 \) in (3.34) and using (3.36), (3.38) and (3.41) in the resulting equation, we get
\[ K_2(\infty) = \frac{1}{P_e} - \frac{\sin \mu_0}{\mu_0 \left( 1 + \frac{\sin 2\mu_0}{2\mu_0} \right)} \sum_{j=0}^{9} B_{j,1} I_{j,0} \] (3.43)

where, \( B_{j,1} = -\left( \frac{\mu_j^2 - \mu_0^2}{2} \right)^{-1} \left( 1 + \frac{\sin 2\mu_0}{2\mu_0} \right)^{-1} \mu_0 \sin \mu_0 I(j,0) \)

Using the asymptotic coefficients \( K_0(\infty), K_1(\infty), \) and \( K_2(\infty), \) in (3.9), one can determine the mean concentration distribution as a function of \( X, \tau \) and the parameters \( a \) and \( \beta. \)

The initial condition for solving (3.9) can be obtained from (2.22) by taking the cross-sectional average. Since we are making long time evaluations of the coefficients, we note that the side effect is independent of \( \theta_m \) on the initial concentration distribution. The solution of (3.9) with asymptotic coefficients can be written as:
\[ \theta_m(\tau, X) = \frac{1}{2P_e \sqrt{\pi K_2(\infty) \tau}} \exp \left[ K_0(\infty) \tau - \frac{[X + K_1(\infty) \tau]^2}{4K_2(\infty) \tau} \right] \] (3.44)

where,
\[ \theta_m(\tau, \infty) = 0, \quad \frac{\partial \theta_m}{\partial X}(\tau, \infty) = 0 \]

4. Results and Discussions

We have modeled the solvent as a couple stress fluid (blood) and studied dispersion of solute in a blood flow bounded by porous beds in the presence of magnetic field considering heterogeneous chemical reaction, on the interphase. The walls of the channel act as catalysts to the reaction. The problem brings into focus three important dispersion coefficients namely, the exchange coefficient \(-K_0\) which arises essentially due to the wall reaction, the convective coefficient \(-K_1\) and diffusive coefficient \(K_2.\) The asymptotic values of these three coefficient are plotted in figures 2 to 16 for different value of Hartmann number \((M < 1, M = 1, M > 1),\) Couple Stress Parameter \((a = 5, 10, 20),\) reaction rate Parameter \((\beta = 10^{-2}, 1, 10^2)\) and Porous Parameter \((\sigma = 100, 200, 500).\) This paper gives the solutions in MATHEMATICA 8.0.

From figure 2, it is clear that \(-K_0\) increases with an increase in the wall reaction parameter \(\beta\) but it is unaffected by the couple stress parameter \(a,\) Hartmann number \((M)\) and porous parameter \((\sigma).\) The classical convective coefficient \((-K_1)\) is plotted in figures 3 to 5 versus wall reaction parameter \(\beta\) for different values of couple stress parameter \(a,\) Hartmann number \((M)\) and porous parameter \((\sigma)\) respectively for a fixed value of slip parameter \(\alpha = 0.1, \beta_1 = 0.1, P_e = 100\) and \(h = 2.\) From figure 3 and 4,
we conclude that $-K_1$ decrease, with increasing values of couple stress parameter and Hartmann number. Figure 5 shows that increase in ($-K_1$), increase the value of ($\sigma$). This is advantageous is maintaining the laminar flow.

The scaled dispersion coefficient $K_2 - P^{-2}_s$ is plotted versus $\beta$ in figures 6 to 8 for different values of $a, M$ and $\sigma$. Figure 6 and 7, show that the increase in $a$ and $M$, the effective dispersion coefficient decreases. From figure 8, we conclude that, increase in $\sigma$ is to increase the effective dispersion coefficient $K_2$. These are useful in the control of dispersion of a solute.

The cross sectional average concentration $\theta_m$ is plotted versus $X$ in figures 9 to 12 respectively for different values of $a, M, \sigma$ and $\beta$ and for fixed values of other parameters given in these figures. It is shows that increase in $\beta$ and $\sigma$ decreases $\theta_m$, while an increase in ($a$) and ($M$) increases $\theta_m$ as expected on the physical ground. This may be attributed to increase in $\sigma$ and $\beta$ is to reduce the velocity and hence to decrease $\theta_m$.

The cross sectional average concentration $\theta_m$ is also plotted against the time $\tau$ in Figures 13 to 16 respectively for different values of $a, M, \sigma$ and $\beta$ and for fixed values of other parameters given in these figures. We conclude that the peak of $\theta_m$ decreases with an increase in $\beta$ occurring at the lower interval of time $\tau$. We also note that, although the peak decreases with an increase in $\sigma$ and increases with an increase in $a$ and $M$, but occurs at almost at the same interval of time $\tau$. These results are useful to understand the transport of solute at different times.

5. Conclusion

The present investigation brings out some interesting results on the dispersion process in flows of blood modeled as couple stress fluid in the presence of magnetic field. The convective dispersion process is analyzed employing the dispersion model of Gill and Sankarasubramanian [8]. It is observed that the effect of magnetic field on dispersion coefficient is found to decrease with increase in $\beta$ and cross sectional average concentration is to increase the time to reach its peak value.

References


Table 1. Roots of the equation $\mu_n \tan \mu_n = \beta$

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![Figure 2. Plots of exchange coefficient versus reaction rate parameter $\beta$](image)

Figure 2. Plots of exchange coefficient versus reaction rate parameter $\beta$

![Figure 3. Plots of convective coefficient $-K_1$ versus $\beta$ for different values of $a$ when $M = 1$ and $\sigma = 100$](image)

Figure 3. Plots of convective coefficient $-K_1$ versus $\beta$ for different values of $a$ when $M = 1$ and $\sigma = 100$

![Figure 4. Plots of convective coefficient $-K_1$ versus $\beta$ for different values of $M$ when $\sigma = 100$ and $a = 5$](image)

Figure 4. Plots of convective coefficient $-K_1$ versus $\beta$ for different values of $M$ when $\sigma = 100$ and $a = 5$
Figure 5. Plots of convective coefficient \(-K_1\) versus \(\beta\) for different values of \(\sigma\) when \(M = 1\) and \(a = 5\)

Figure 6. Plots of scale dispersion coefficient \(K_2 - Pe^{-2}\) versus \(\beta\) for different values of \(a\) when \(M = 1\) and \(\sigma = 100\)

Figure 7. Plots of scale dispersion coefficient \(K_2 - Pe^{-2}\) versus \(\beta\) for different values of \(M\) when \(\sigma = 100\) and \(a = 5\)
Figure 8. Plots of scale dispersion coefficient $K_2 - Pe^{-2}$ versus $\beta$ for different values of $\sigma$ when $M = 1$ and $a = 5$

Figure 9. Plots of mean concentration $\theta_m$ versus $X$ for different values of $a$ when $M = 1, \sigma = 100, \beta = 10^{-2}$ and $\tau = 0.6$

Figure 10. Plots of mean concentration $\theta_m$ versus $X$ for different values of $M$ when $a = 5, \sigma = 100, \beta = 10^{-2}$ and $\tau = 0.6$
Figure 11. Plots of mean concentration $\theta_m$ versus $X$ for different values of $\sigma$ when $M = 1, \alpha = 5, \beta = 10^{-2}$ and $\tau = 0.6$.

Figure 12. Plots of mean concentration $\theta_m$ versus $X$ for different values of $\beta$ when $M = 1, \alpha = 5, \sigma = 100$ and $\tau = 0.6$.

Figure 13. Plots of mean concentration $\theta_m$ versus $\tau$ for different values of $\alpha$ when $M = 1, \sigma = 100, \beta = 10^{-2}$ and $X = 0.8$. 
Figure 14. Plots of mean concentration $\theta_m$ versus $\tau$ for different values of $M$ when $a = 5, \sigma = 100, \beta = 10^{-2}$ and $X = 0.8$

Figure 15. Plots of mean concentration $\theta_m$ versus $\tau$ for different values of $\sigma$ when $M = 1, a = 5, \beta = 10^{-2}$ and $X = 0.8$

Figure 16. Plots of mean concentration $\theta_m$ versus $\tau$ for different values of $\beta$ when $M = 1, a = 5, \sigma = 100$ and $X = 0.8$

APPENDIX A
\[ m_1 = \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}, \quad Eq.(A.1) \]
\[ m_3 = \frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}, \quad Eq.(A.2) \]
\[ a_3 = (m_1 + \alpha \sigma)e^{m_1}, \quad Eq.(A.3) \]
\[ a_4 = (m_1 - \alpha \sigma)e^{-m_1}, \quad Eq.(A.4) \]
\[ a_5 = (m_3 + \alpha \sigma)e^{m_3}, \quad Eq.(A.5) \]
\[ a_6 = (m_3 - \alpha \sigma)e^{-m_3}, \quad Eq.(A.6) \]
\[ a_7 = \left( \frac{P}{M^2} - \frac{k}{\mu (1 + \beta_1)} \frac{\partial p}{\partial x} \right) \alpha \sigma, \quad Eq.(A.7) \]
\[ a_8 = m_1^2 e^{m_1}, a_9 = m_1^2 e^{-m_1}, \quad Eq.(A.8) \]
\[ a_{10} = m_3^2 e^{m_3}, a_{11} = m_3^2 e^{-m_3}, \quad Eq.(A.9) \]
\[ C_1 = C_2 = \frac{a_{5a_8} - a_{6a_8} + a_{5a_9} - a_{6a_9} - a_{3a_{10}} + a_{4a_{10}} - a_{3a_{11}} + a_{4a_{11}}}{a_{7a_8} + a_{7a_9}}, \quad Eq.(A.10) \]
\[ C_3 = C_4 = \frac{a_{5a_8} - a_{6a_8} + a_{5a_9} - a_{6a_9} - a_{3a_{10}} + a_{4a_{10}} - a_{3a_{11}} + a_{4a_{11}}}{a_{7a_8} + a_{7a_9}}, \quad Eq.(A.11) \]
\[ A_1 = \frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{P}{M^2}, \quad Eq.(A.12) \]
\[ A_2 = \frac{C_1 \sinh m_1}{m_1} + \frac{C_3 \sinh m_3}{m_3}, \quad Eq.(A.13) \]
\[ A_3 = \frac{C_1 \sinh m_1}{m_1^3} + \frac{C_3 \sinh m_3}{m_3^3}. \quad Eq.(A.14) \]