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EXACT ANALYSIS OF UNSTEADY CONVECTIVE DIFFUSION FOR BLOOD FLOW WITH INTERPHASE MASS TRANSFER IN MAGNETIC FIELD

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Abstract This paper deals with a mathematical model for exact analysis of miscible dispersion of solute with interphase mass transfer in a blood(couple stress fluid) flow bounded by porous beds under the influence of magnetic field. The three coefficients, namely, exchange coefficient, convection coefficient, and dispersion coefficient are evaluated asymptotically at large-time using generalized dispersion model. The dispersion equation is used to calculate the mean concentration distribution of a solute, bounded by the porous layer, and is expressed as a function of dimensionless axial distance and time. It is computed for different values of Hartmann number (M), Couple Stress Parameter(a), reaction rate Parameter(β) and Porous Parameter (σ).

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1. Introduction

In many biomedical problems, the interphase mass transfer plays an important role because many physiological situation involve interphase mass transfer. Therefore, it is necessary to develop a technique for handling such problems, which involve interphase mass transport. Several authors have studied various characteristics of dispersion were mainly concerned with Taylors dispersion [23], which is valid for large time. Physiological fluid flow problems have been mainly concerned with transient phenomena where Taylors model is not valid. However, Sankarasubramanian and Gill[18] have developed an analytical method to analyze transient dispersion of non-uniform initial distribution called

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generalized dispersion in laminar flow in a tube with a first order chemical reaction at the tube wall. This method can be applied to physiological problems, where a first order chemical reaction occurs at the tube wall. One such situation is transport of oxygen and nutrients to tissue cells and removal of metabolic waste products from tissue cells. Interphase mass transfer also takes places in pulmonary capillaries, where the carbon dioxide is removed from the blood and oxygen is taken up by the blood.

Rudraiah et al.[17] studied the dispersion in a stokes couple-stress fluid flow by using the generalized model of Gill and Sankarasubramanian [8]. Considering solute reaction at the channel walls in their all time analysis of dispersion, reaction at the walls is of practical interest and in the simplest case, a first order chemical reaction at the walls is considered by them, in carrying out an exact analysis of unsteady convection in couple stress fluid flows.

Gill [7] developed a local theory of Taylor diffusion in fully developed laminar tube flow with a periodic condition at the inlet of the tube. Gill [8] obtained an exact solution to the unsteady convective diffusion equation for miscible displacement in fully developed laminar flow in tubes by defining dispersion coefficient to be functions of time. Shivakumar et al. [20] have obtained a closed-form solution for unsteady convective diffusion in a fluid-saturated sparsely packed porous medium using the generalized dispersion model of [8]. In all these investigations, it is assumed that, the solute does not chemically react in the liquid in which it is dispersed. Gupta [9], following Taylor [23], studied the effects of homogeneous and heterogeneous reaction on the dispersion of a solute in the laminar flow of Newtonian fluid between two parallel plates.

Shukla et al.[19], Soundalgekar [21], Meena Priya[10], Dulal Pal [3] and Dutta et al. [4] studied dispersion in non-Newtonian fluids by considering only homogeneous first-order chemical reaction in the bulk of the fluid. Chandra and Agarwal [2] considered dispersion in simple microfluid flows taking only homogeneous reaction into consideration. Suvadip Paul [22] examine a the influence of angularity on the transport process under the combined effects of reversible and irreversible wall reactions, when the flow is driven by a pressure gradient comprising of steady and periodic components. Francesco Gentile [5] studied the transport formulation proposed in [6] was further developed to account for the time dependency of the problem. Prathap Kumar et al.[11] also investigated the effect of homogeneous and heterogeneous reactions on the solute dispersion in composite porous medium. Recently,Ramana Rao[13], Ramana[14,15] and Ramana et al [16] studied combined the effect of non-Newtonian rheology and irreversible boundary reaction on dispersion in a Herschel-Bulkley fluid through a conduit, (pipe/channel) by using the generalized dispersion model proposed by [8]. Pauling [12] first reported that the erythrocytes orient with their disk plane parallel to the magnetic field.

This paper deals with the effect of couple stress and magnetic field on the unsteady convective diffusion, with interphase mass transfer by using the generalized dispersion model of Sankarasubramanian and Gill [18]. Convection coefficient K_1 and Dispersion coefficient K_2 are influenced by the couple stress parameter arising due to suspension in the fluid, magnetic field and porous parameter. The exchange coefficient K_0 arises mainly due to the interphase mass transfer, and it is independent of the solvent fluid velocity. The interphase mass transfer also influence the convection and dispersion coefficients.

2. Mathematical Formulation

We have considered a steady laminar and fully developed flow (unidirectional) in a channel bounded by porous layers and separated by a distance $2h$. A schematic diagram of the physical configuration and the description of the initial slug input of concentration are shown in figure 1. A uniform magnetic field B_o is applied in the y-direction to the flow of blood. We make the following assumption for electromagnetic interactions, i) the induce magnetic field and the electric field produced by the motion of blood are negligible (since blood has low magnetic Reynolds number). ii) no external electric field is applied. Flow region is divided into two sub-region such as Fluid film region and Porous tissue region. The governing equation of the motion for flow in vector form is given by

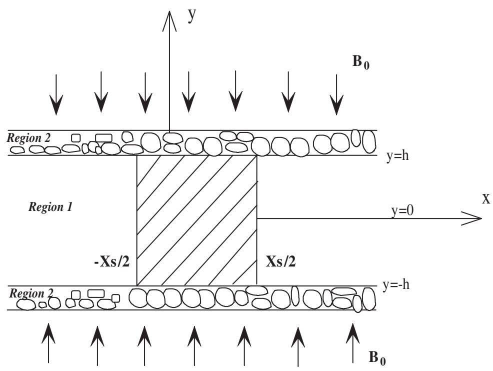

Figure 1. Physical Configuration

Region 1:**Fluid Film Region**
Conservation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

Conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} - \lambda\nabla^4\vec{q} + J\times B \tag{2.2}$$

Region 2:**Porous Tissue Region**
Conservation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0 \tag{2.3}$$

Conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} - \frac{\mu}{k}(1+\beta_1)\vec{q} \tag{2.4}$$

Maxwell's equations are

$$\nabla \cdot B = 0, \nabla \times \cdot B = \mu_0 J$$

$$\nabla \times E = -\frac{\partial B_0}{\partial t}$$

Ohm's law

$$J = \sigma_0(E + \vec{q}\times B_0)$$

Conservation of species

$$\frac{\partial \vec{C}}{\partial t} + (\vec{q} \cdot \nabla)C = D\nabla^2 C \tag{2.5}$$

In cartesian form, using the above equation (2.1)-(2.4) becomes
Region 1:**Fluid Film Region**

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \lambda \frac{\partial^4 u}{\partial y^4} - B_0^2 \sigma_0 u, \tag{2.6}$$

$$0 = -\frac{\partial p}{\partial y} \tag{2.7}$$

Region 2:**Porous Tissue Region**

$$0 = -\frac{\partial p}{\partial x} - \frac{\mu}{k}(1+\beta_1)u_p, \tag{2.8}$$

$$0 = -\frac{\partial p}{\partial y} \tag{2.9}$$

where, $\vec{q} = \vec{i}u + \vec{j}v$, u is the x components of velocity, p is the pressure, ρ is the density of the fluid, μ is the viscosity of the fluid, λ is the couple stress parameter, k is the permeability of the porous medium, E the electric field,σ_0 is the electrical conductivity, J the current density and u_p is the darcy velocity. It may be noted that, (2.8) is the modified darcy equation. where, β_1 is the couple stress parameter.

We consider the dispersion of reactive solute in the fully developed flow through a parallel plate channel bounded by porous beds. Introduced a slug of concentration $C = C_0\psi_1(x)Y_1(y)$. The mass balance equation (2.5) concerning the solute concentration C undergoes heterogeneous chemical reaction such as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{2.10}$$

where, D is the molecular diffusivity.
The boundary conditions on velocity are

$$\frac{\partial u}{\partial y} = \frac{-\alpha}{\sqrt{k}}(u - u_p) \ at \ y = h, \tag{2.11}$$

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k}}(u - u_p) \ at \ y = -h, \tag{2.12}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \ at \ y = \pm h. \tag{2.13}$$

where, α is the slip parameter. Eqs. (2.11) and (2.12) is Beavers and Joseph [1] slip condition at the lower and upper permeable surfaces and equation (2.13) specifies the vanishing of the couple stress.

Initial and Boundary conditions on concentration

The initial distribution assumed to be in a variable separable form is given by

$$C(0, x, y) = C_0\psi_1(x)Y_1(y), \tag{2.14}$$

The heterogeneous reaction conditions are:

$$\left.\begin{array}{rcll} -D\frac{\partial C}{\partial y} & = & K_s C & at \ \ y = h \ and \\ D\frac{\partial C}{\partial y} & = & K_s C & at \ \ y = -h \end{array}\right\} \tag{2.15}$$

where, K_s is the reaction rate constant catalysed by the walls and C_0 is a reference concentration.
As the amount of solute in the system is finite,

$$C(t,\infty,y) = \frac{\partial C}{\partial x}(t,\infty,y) = 0 \tag{2.16}$$

where, C_0 is reference concentration.
Now, we introduce the non-dimensional quantities,

$$U = \frac{u}{\bar{u}};\quad \eta = \frac{y}{h};\ \theta = \frac{C}{C_0};\quad X = \frac{Dx}{h^2\bar{u}};\quad X_s = \frac{Dx_s}{h^2\bar{u}};\quad \tau = \frac{Dt}{h^2};\quad Pe = \frac{h\bar{u}}{D};$$

Equation (2.6) and (2.10) in non-dimensional form as

$$\frac{\partial^4 U}{\partial \eta^4} - a^2\frac{\partial^2 U}{\partial \eta^2} + a^2M^2U = Pa^2 \tag{2.17}$$

and

$$\frac{\partial \theta}{\partial \tau} + U^*\frac{\partial \theta}{\partial X_1} = \frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial X_1^2} + \frac{\partial^2 \theta}{\partial \eta^2} \tag{2.18}$$

where, $P = \frac{-h^2}{\mu}\frac{\partial p}{\partial x}, l = \sqrt{\frac{\lambda}{\mu}}$ and $a = \frac{h}{l}$ is the couple stress parameter, $M^2 = \frac{B_0^2\sigma_0 h^2}{\mu}$ is the square of the Hartmann number, $Pe = \frac{\bar{u}h}{D}$ is the peclet number, $U^* = \frac{U-\bar{U}}{\bar{U}}$ is normalized axial component of velocity as $x_1 = x - \bar{u}t$ which is dimensionless form is $X_1 = X - \tau$ where $X_1 = \frac{x_1 D}{h^2 a}$
The initial and boundary conditions of (2.11) to (2.16) in dimensionless form

$$\frac{\partial U}{\partial \eta} = -\alpha\sigma(U - U_p)\ at\ \eta = 1 \tag{2.19}$$

$$\frac{\partial U}{\partial \eta} = \alpha\sigma(U - U_p)\ at\ \eta = -1 \tag{2.20}$$

$$\frac{\partial^2 U}{\partial \eta^2} = 0\ at\ y = \pm 1 \tag{2.21}$$

where, $\sigma = \frac{h}{\sqrt{k}}$ is the porous parameter.

$$\theta(0,X,\eta) = \psi(X)Y(\eta), \tag{2.22}$$

$$\left.\begin{aligned} \frac{\partial \theta}{\partial \eta} &= -\beta\theta \quad at\ \eta = 1, \\ \frac{\partial \theta}{\partial \eta} &= \beta\theta \quad at\ \eta = -1 \end{aligned}\right\} \tag{2.23}$$

$$\theta(\tau,\infty,\eta) = \frac{\partial \theta}{\partial X}(\tau,\infty,\eta) = 0 \tag{2.24}$$

3. Method of solution

Velocity distribution

The solution of Equation (2.17) can be written as:

$$U(\eta) = C_1 e^{m_1\eta} + C_2 e^{-m_1\eta} + C_3 e^{m_3\eta} + C_4 e^{-m_3\eta} + \frac{P}{M^2} \tag{3.1}$$

where, C_1,C_2,C_3 and C_4 are constants. Applying the boundary conditions (2.19)-(2.21) in (3.1), we obtain the the velocity of blood as

$$U(\eta) = 2C_1 Coshm_1\eta + 2C_3 Coshm_3\eta + \frac{P}{M^2} \tag{3.2}$$

The normalized axial components of velocity obtained from(3.1) is

$$U^* = \frac{U - \bar{U}}{\bar{U}} = \frac{2}{A_1}\left[C_1 Coshm_1\eta + C_3 Coshm_3\eta - \left(\frac{C_1 Sinhm_1}{m_1} + \frac{C_3 Sinhm_3}{m_3}\right)\right] \tag{3.3}$$

where,

$$\bar{U} = \frac{1}{2}\int_{-1}^{1} U(\eta)d\eta = \frac{2C_1 Sinhm_1}{m_1} + \frac{2C_3 Sinhm_3}{m_3} + \frac{P}{M^2} \tag{3.4}$$

Generalized Dispersion model

The solution of (2.18) subject to the conditions (2.22)-(2.24), following Gill and Sankarasubramanian [18] is

$$\theta(\tau, X, \eta) = \sum_{k=0}^{\infty} f_k(\tau,\eta)\frac{\partial^k \theta_m}{\partial X^k}, \tag{3.5}$$

where, θ_m is the dimensionless cross sectional average concentration and is given by

$$\theta_m = \frac{1}{2}\int_{-1}^{1} \theta(\tau, X, \eta)d\eta \tag{3.6}$$

Equation (2.18) is multiplied throughout by $\frac{1}{2}$ and integrated with respect to y between the limits -1 to 1 and using (3.6) we get,

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2}\frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2}\left[\frac{\partial \theta}{\partial \eta}\right]_{-1}^{1} - \frac{1}{2}\frac{\partial}{\partial X}\int_{-1}^{1} U^*\theta d\eta \tag{3.7}$$

Using Eq. (3.5) in (3.7), we get the dispersion model for θ_m as

$$\begin{aligned}\frac{\partial \theta_m}{\partial \tau} &= \frac{1}{P_e^2}\frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2}\left[\frac{\partial}{\partial \eta}(f_0(\tau,\eta)\theta_m(\tau,X) + f_1(\tau,\eta)\frac{\partial \theta_m}{\partial X}(\tau,X) + \ldots)\right]_{-1}^{1} \\ &\quad -\frac{1}{2}\frac{\partial}{\partial X}\int_{-1}^{1} U^*(f_0(\tau,\eta)\theta_m(\tau,X) \\ &\quad +f_1(\tau,\eta)\frac{\partial \theta_m}{\partial X}(\tau,X) + f_2(\tau,\eta)\frac{\partial^2 \theta_m}{\partial X^2}(\tau,X) + \ldots)d\eta \end{aligned} \tag{3.8}$$

The generalized dispersion model of [7] is defined as

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{i=0}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial X^i} \tag{3.9}$$

substituting (3.9) in (3.8) we get,

$$\begin{aligned} K_0\theta_m + K_1\frac{\partial \theta_m}{\partial X} + K_2\frac{\partial^2 \theta_m}{\partial X^2} &= \frac{1}{P_e^2}\frac{\partial^2 \theta_m}{\partial X^2} + \frac{1}{2}\left[\frac{\partial}{\partial \eta}(f_0\theta_m + f_1\frac{\partial \theta_m}{\partial X} + \ldots)\right]_{-1}^{1} \\ & -\frac{1}{2}\frac{\partial}{\partial X}\int_{-1}^{1} U^*\left(f_0(\tau,\xi)\theta_m(\tau,X) + f_1(\tau,\xi)\frac{\partial \theta_m}{\partial X}(\tau,X)\right. \\ & \left. +f_2(\tau,\xi)\frac{\partial^2 \theta_m}{\partial X^2}(\tau,X) + \ldots\right) d\eta \end{aligned}$$

Equating like coefficient of $\theta_m, \frac{\partial \theta_m}{\partial X}, \frac{\partial^2 \theta_m}{\partial X^2} \ldots$ we get K_i's are,

$$K_i(\tau) = \frac{\delta_{i2}}{P_e^2} + \frac{1}{2}\frac{\partial f_i}{\partial \eta}(\tau,1) - \frac{1}{2}\int_{-1}^{1} f_{i-1}(\tau,\eta)U^*(\tau,\eta)d\eta \tag{3.10}$$

$$(i = 1, 2, 3, \ldots)$$

where, $f_{-1} = 0$ and δ_{i2} is the Kroneckar delta defined by

$$\delta_{i2} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

The exchange coefficient $K_0(\tau)$ accounts for the non-zero solute flux at the channel wall, and negative sign indicates the depletion of solute in the system with time caused by the irreversible reaction, which occurs at the channel wall. The presence of non-zero solute flux at the walls of the channel, also affects the higher order K_i due to the explicit appearance of $\frac{\partial f_i}{\partial \eta}(\tau, 1)$ in equation (3.10). Equation (3.9) can be truncated after the term involving K_2 without causing serious error, because K_3, K_4, etc. become negligibly small compared to K_2. The resulting model for the mean concentration is

$$\frac{\partial \theta_m}{\partial \tau} = K_0(\tau)\theta_m + K_1(\tau)\frac{\partial \theta_m}{\partial X} + K_2(\tau)\frac{\partial^2 \theta_m}{\partial X^2} \tag{3.11}$$

To solve the equation (3.11), we need the coefficients $K_i(\tau)$ in addition to the appropriate initial and boundary conditions. For this, the corresponding function f_k must be determined. So, substituting (3.5) into (2.18), the following set of differential equations for f_k are generated.

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial \eta^2} - U^* f_{k-1} + \frac{1}{P_e^2} f_{k-2} - \sum_{i=0}^{k} K_i f_{k-i}, \quad (k = 0, 1, 2, \ldots) \tag{3.12}$$

where, $f_{-1} = f_{-2} = 0$.

To evaluate $K_i's$, we need to know the f_k's which are obtained by solving (3.12) for f_k's subject to the boundary conditions,

$$f_k(\tau, 0) = finite, \tag{3.13}$$

$$\frac{\partial f_k}{\partial \eta}(\tau, 1) = -\beta f_k(\tau, 1), \tag{3.14}$$

$$\frac{\partial f_k}{\partial \eta}(\tau, 0) = 0, \tag{3.15}$$

$$\frac{1}{2}\int_{-1}^{1} f_k(\tau, \eta)d\eta = \delta_{k_0}, (k = 0, 1, 2) \tag{3.16}$$

The function f_0 and the exchange coefficient K_0 are independent of the velocity and can be solved easily. It should be pointed out here, that, a simultaneous solution has to be obtained from these two quantities since K_0, which can be obtained from (3.10) as

$$K_0(\tau) = \frac{1}{2}\left[\frac{\partial f_0}{\partial \eta}\right]_{-1}^{1} \tag{3.17}$$

Substituting $k = 0$ in equation (3.12) we get the differential equation for f_0 as

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial \eta^2} - f_0 K_0 \tag{3.18}$$

We derive an initial condition for f_0 using (3.6) by taking $\tau = 0$ in that equation to get

$$\theta_m(0, X) = \frac{1}{2}\int_{-1}^{1} \theta(0, X, \eta)d\eta \tag{3.19}$$

Substituting $\tau = 0$ in (3.5) and setting $f_k(\eta) = 0(k = 1, 2, 3)$ gives us the initial condition for f_0 as

$$f_0(0, \eta) = \frac{\theta(0, X, \eta)}{\theta_m(0, X)} \tag{3.20}$$

We note that the left hand side of (3.20) is a function of η only and the right hand side is a function of both X and η. Thus, clearly the initial concentration distribution must be a separable function of X and η. Substituting equation (2.14) and (3.19) into equation (3.20), we get

$$f_0(0, \eta) = \frac{\psi(\eta)}{\frac{1}{2}\int_{-1}^{1} \psi(\eta)d\eta} \tag{3.21}$$

The solution of the reaction diffusion equation (3.18) with these conditions may be formulated as

$$f_0(\tau, \eta) = g_0(\tau, \eta) \;\; exp \left[-\int_{0}^{\tau} K_0(\eta)d\eta\right] \tag{3.22}$$

from which it follows that $g_0(\tau, \eta)$ has to satisfy

$$\frac{\partial g_0}{\partial \tau} = \frac{\partial^2 g_0}{\partial \eta^2} \tag{3.23}$$

with conditions

$$f_0(0,\eta) = g_0(0,\eta) = \frac{\psi(\eta)}{\frac{1}{2}\int_{-1}^{1}\psi(\eta)d\eta}, \tag{3.24}$$

$$g_0(\tau,0) = finite, \tag{3.25}$$

$$\frac{\partial g_0}{\partial \eta}(\tau,1) = -\beta g_0(\tau,1). \tag{3.26}$$

The solution of (3.23) subject to conditions (3.24)-(3.26) is

$$g_0(\tau,\eta) = \sum_{n=0}^{\infty} A_n \quad Cos(\mu_n\eta)exp[-\mu_n^2\tau] \tag{3.27}$$

where, μ_n's are the roots of

$$\mu_n tan\mu_n = \beta, \quad n = 0,1,2,\ldots \tag{3.28}$$

and A_n's are given by

$$A_n = \frac{2\int_{-1}^{1}\psi(\eta)Cos\mu_n\eta d\eta}{\left(1+\frac{Sin2\mu_n}{2\mu_n}\right)\int_{-1}^{1}\psi(\eta)d\eta} \tag{3.29}$$

from (3.22), it follows that

$$f_0(\tau,\eta) = \frac{2g_0(\tau,\eta)}{\int_{-1}^{1} g_0(\tau,\eta)d\eta} = \frac{\sum_{n=0}^{9} A_n exp[-\mu_n^2\tau]Cos\mu_n\eta}{\sum_{n=0}^{9}\frac{A_n}{\mu_n}exp[-\mu_n^2\tau]Sin\mu_n} \tag{3.30}$$

The first ten roots of the transcendental equation (3.28) are obtained using MATHEMATICA 8.0 and are given in Table 1. We find that these ten roots ensured convergence of the series seen in the expansions for f_0 and K_0. Having obtained f_0, we get K_0 from (3.17) in the form

$$K_0(\tau) = -\frac{\sum_{n=0}^{9} A_n\mu_n exp[-\mu_n^2\tau]Sin\mu_n}{\sum_{n=0}^{9}\frac{A_n}{\mu_n}exp[-\mu_n^2\tau]Sin\mu_n} \tag{3.31}$$

$K_0(\tau)$ is independent of velocity distribution.

As $\tau\to\infty$, we get the asymptotic solution for K_0 from (3.31) as

$$K_0(\infty) = -\mu_0^2 \tag{3.32}$$

where, μ_0 is the first root of the equation (3.28). Physically, this represents first order chemical reaction coefficient having obtained $K_0(\infty)$. We can now get $K_1(\infty)$, from (3.10) (with $i = 1$) knowing $f_0(\infty,\eta)$ and $f_1(\infty,\eta)$. Likewise, $K_2(\infty)$, $K_3(\infty),\ldots$ require the knowledge of K_0, K_1, f_0, f_1 and f_2. Equation (3.30) in the limit $\tau\to\infty$ reduces to

$$f_0(\infty,\eta) = \frac{\mu_0}{Sin\mu_0}Cos(\mu_0\eta) \tag{3.33}$$

Then we find f_1, K_1, f_2, and K_2. For asymptotically long times, i.e., $\tau \to \infty$, equation (3.10) and (3.12) give us K_i's and f_k's as

$$K_i(\infty) = \frac{\delta_{i2}}{P_e^2} - \beta f_i(\infty, 1) - \int_{-1}^{1} U^* f_{i-1}(\infty, \eta) d\eta, \quad (i = 1, 2, 3) \tag{3.34}$$

$$\frac{d^2 f_k}{d\eta^2} + \mu_0^2 f_k = (U^* + K_1) f_{k-1} - \left(\frac{1}{P_e^2} - K_2\right) f_{k-2}, \quad (k = 1, 2) \tag{3.35}$$

The f_k's must satisfy the conditions (3.6) and this permits the eigen function expansion in the form of

$$f_k(\infty, \eta) = \sum_{j=0}^{9} B_{j,k} Cos(\mu_j \eta), \quad k = 1, 2, 3, \dots \tag{3.36}$$

Substituting (3.36) in (3.35) and multiplying the resulting equation by $Cos(\mu_j \eta)$ and integrating with respect to η from -1 to 1,

$$B_{j,k} Cos \mu_j \eta = \frac{1}{\mu_j^2 - \mu_0^2} \left[\frac{1}{P_e^2} \sum_{j=0}^{\infty} B_{j,k-2} Cos \mu_j \eta \right.$$
$$\left. - U^* \sum_{j=0}^{\infty} B_{j,k-1} Cos \mu_j \eta - \sum_{j=0}^{\infty} K_i B_{j,k-i} Cos \mu_j \eta \right]$$

multiplying by $Cos \mu_l \eta$ and integrating with respect to η, we get

$$B_{j,k} = \frac{1}{(\mu_j^2 - \mu_0^2)} \left[\frac{1}{P_e^2} B_{j,k-2} - \sum_{i=1}^{k} K_i B_{j,k-i} - \left(1 + \frac{Sin 2\mu_j}{2\mu_j}\right)^{-1} \sum_{j=0}^{9} B_{j,k-1} I(j,l) \right]$$
$$k = (1, 2) \tag{3.37}$$

where,

$$I(j,l) = \int_{-1}^{1} U^* Cos \mu_j \eta Cos \mu_l \eta d\eta = I(l,j) \tag{3.38}$$

$$B_{j,-1} = 0, B_{j,0} = 0 \quad for \quad j = 1 \ to \ 9 \tag{3.39}$$

The first expansion coefficient $B_{0,k}$ in equation (3.36) using conditions (3.13)-(3.16) can be expressed in terms of $B_{j,k}$ $(j = 1 \ to \ 9)$ as,

$$f_k = B_{0,k} Cos \mu_0 \eta + \sum_{j=1}^{\infty} B_{j,k} Cos \mu_j \eta \quad \text{(from equation 3.36)}$$

By integrating this equation, we get

$$0 = B_{0,k} \frac{Sin \mu_0 \eta}{\mu_0} + \sum_{j=1}^{\infty} B_{j,k} \frac{Sin \mu_j \eta}{\mu_j}$$

(Using the boundary condition $\int_{-1}^{1} f_k(\tau, \eta) d\eta = \delta_{k0} = 0$)

$$B_{0,k} = -\left(\frac{\mu_0}{Sin \mu_0}\right) \sum_{j=1}^{9} B_{j,k} \frac{Sin \mu_j}{\mu_j}, \quad (k = 1, 2, 3, \dots) \tag{3.40}$$

Further, from (3.36) and (3.33) we find that

$$B_{0,0} = \frac{\mu_0}{Sin\mu_0} \tag{3.41}$$

Substituting $i = 1$ in (3.34) and using (3.38), (3.39) and (3.41) in the resulting equation, we get

$$K_1(\infty) = -\frac{I(0,0)}{\left[1 + \frac{Sin2\mu_0}{2\mu_0}\right]} \tag{3.42}$$

Substituting $i = 2$ in (3.34) and using (3.36), (3.38) and (3.41) in the resulting equation, we get

$$K_2 = \frac{1}{P_e^2} - \frac{Sin\mu_0}{\mu_0\left(1 + \frac{Sin2\mu_0}{2\mu_0}\right)}\sum_{j=0}^{9} B_{j,1} I_{j,0} \tag{3.43}$$

where, $B_{j,1} = -(\mu_j^2 - \mu_0^2)^{-1}\left(1 + \frac{Sin2\mu_0}{2\mu_0}\right)^{-1}\frac{\mu_0}{Sin\mu_0} I(j,0)$

Using the asymptotic coefficients $K_0(\infty)$, $K_1(\infty)$, and $K_2(\infty)$, in (3.9), one can determine the mean concentration distribution as a function of X, τ and the parameters a and β.

The initial condition for solving (3.9) can be obtained from (2.22) by taking the cross-sectional average. Since we are making long time evaluations of the coefficients, we note that the side effect is independent of θ_m on the initial concentration distribution. The solution of (3.9) with asymptotic coefficients can be written as:

$$\theta_m(\tau, X) = \frac{1}{2P_e\sqrt{\pi K_2(\infty)\tau}} exp\left[K_0(\infty)\tau - \frac{[X + K_1(\infty)\tau]^2}{4K_2(\infty)\tau}\right] \tag{3.44}$$

where,

$$\theta_m(\tau, \infty) = 0, \quad \frac{\partial\theta_m}{\partial X}(\tau, \infty) = 0$$

4. Results and Discussions

We have modeled the solvent as a couple stress fluid(blood) and studied dispersion of solute in a blood flow bounded by porous beds in the presence of magnetic field considering heterogeneous chemical reaction, on the interphase. The walls of the channel act as catalysts to the reaction. The problem brings into focus three important dispersion coefficients namely, the exchange coefficient $-K_0$ which arises essentially due to the wall reaction, the convective coefficient $-K_1$ and diffusive coefficient K_2. The asymptotic values of these three coefficient are plotted in figures 2 to 16 for different value of Hartmann number $(M < 1, M = 1, M > 1)$, Couple Stress Parameter$(a = 5, 10, 20)$, reaction rate Parameter$(\beta = 10^{-2}, 1, 10^2)$ and Porous Parameter $(\sigma = 100, 200, 500)$. This paper gives the solutions in MATHEMATICA 8.0.

From figure 2, it is cleat that $-K_0$ increases with an increase in the wall reaction parameter(β) but it is unaffected by the couple stress parameter(a), Hartmann number (M) and porous parameter (σ). The classical convective coefficient$(-K_1)$ is plotted in figures 3 to 5 versus wall reaction parameter (β) for different values of couple stress parameter(a), Hartmann number (M) and porous parameter (σ) respectively for a fixed value of slip parameter $\alpha = 0.1, \beta_1 = 0.1, Pe = 100$ and $h = 2$. From figure 3 and 4, we conclude that $-K_1$ decrease, with increasing values of couple stress parameter and Hartmann number. Figure 5 shows that increase in $(-K_1)$, increase the value of (σ). This is advantageous is maintaining the laminar flow.

The scaled dispersion coefficient $K_2 - P_e^{-2}$ is plotted versus β in figures 6 to 8 for different values of a, M and σ. Figure 6 and 7, show that the increase in a and M, the effective dispersion coefficient decreases. From figure 8, we conclude that, increase in σ is to increase the effective dispersion coefficient K_2. These are useful in the control of dispersion of a solute.

The cross sectional average concentration θ_m is plotted versus X in figures 9 to 12 respectively for different values of a, M, σ and β and for fixed values of other parameters given in these figures. It is shows that increase in β and σ decreases θ_m, while an increase in (a) and (M) increases θ_m as expected on the physical ground. This may be attributed to increase in σ and β is to reduce the velocity and hence to decrease θ_m.

The cross sectional average concentration θ_m is also plotted against the time τ in Figures 13 to 16 respectively for different values of a, M, σ and β and for fixed values of other parameters given in these figures. We conclude that the peak of θ_m decreases with an increase in β occurring at the lower interval of time τ. We also note that, although the peak decreases with an increase in σ and increases with an increase in a and M, but occurs at almost at the same interval of time τ.These results are useful to understand the transport of solute at different times.

5. Conclusion

The present investigation brings out some interesting results on the dispersion process in flows of blood modeled as couple stress fluid in the presence of magnetic field. The convective dispersion process is analyzed employing the dispersion model of Gill and Sankarasubramanian [8]. It is observed that the effect of magnetic field on dispersion coefficient is found to decrease with increase in β and cross sectional average concentration is to increase the time to reach its peak value.

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Table 1. Roots of the equation $\mu_n tan \mu_n = \beta$

β	μ_0	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9
10^{-2}	0.099834	3.14477	6.28478	9.42584	12.5672	15.7086	18.8501	21.9916	25.1331	28.2747
0.05	0.22176	3.15743	6.29113	9.43008	12.5703	15.7111	18.8522	21.9934	25.1347	28.2761
10^{-1}	0.311053	3.1731	6.29906	9.43538	12.5743	15.7143	18.8549	21.9957	25.1367	28.2779
0.5	0.653271	3.29231	6.36162	9.47749	12.606	15.7397	18.876	22.0139	25.1526	28.292
1.0	0.860334	3.42562	6.4373	9.52933	12.6453	15.7713	18.9024	22.2126	25.1724	28.3096
5.0	1.31384	4.03357	6.9096	9.89275	12.9352	16.0107	19.1055	22.2126	25.3276	28.4483
10.0	1.42887	4.3058	7.22811	10.2003	13.2142	16.2594	19.327	22.4108	25.5064	28.6106
100.0	1.55525	4.66577	7.77637	10.8871	13.9981	17.1093	20.2208	23.3327	26.445	29.5577

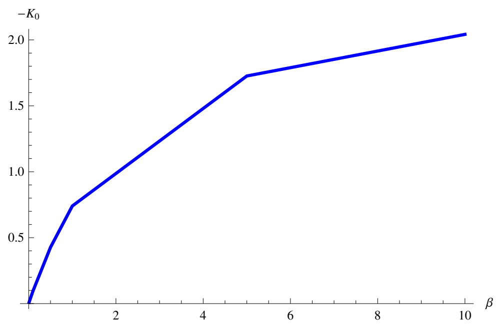

Figure 2. Plots of exchange coefficient versus reaction rate parameter β

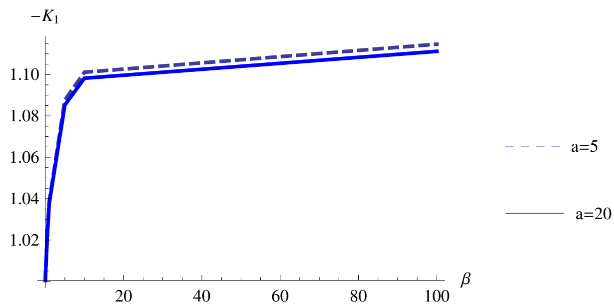

Figure 3. Plots of convective coefficient $-K_1$ versus β for different values of a when $M = 1$ and $\sigma = 100$

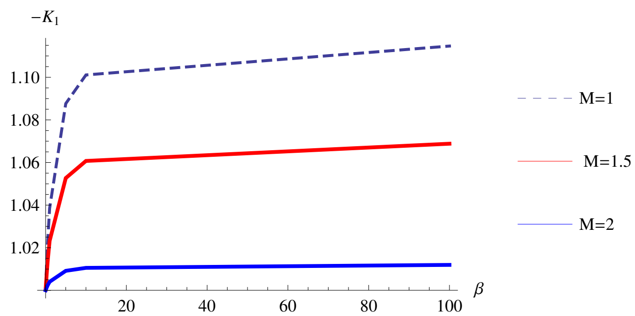

Figure 4. Plots of convective coefficient $-K_1$ versus β for different values of M when $\sigma = 100$ and $a = 5$

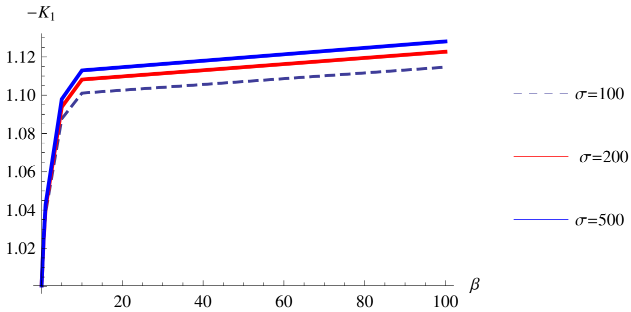

Figure 5. Plots of convective coefficient $-K_1$ versus β for different values of σ when $M = 1$ and $a = 5$

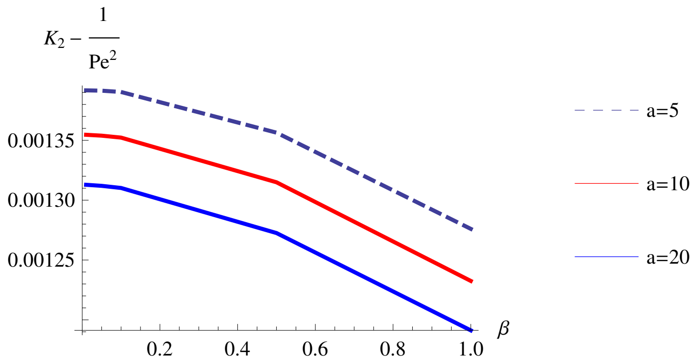

Figure 6. Plots of scale dispersion coefficient $K_2 - Pe^{-2}$ versus β for different values of a when $M = 1$ and $\sigma = 100$

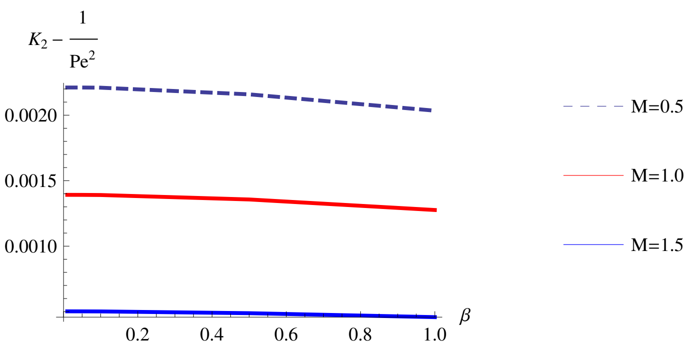

Figure 7. Plots of scale dispersion coefficient $K_2 - Pe^{-2}$ versus β for different values of M when $\sigma = 100$ and $a = 5$

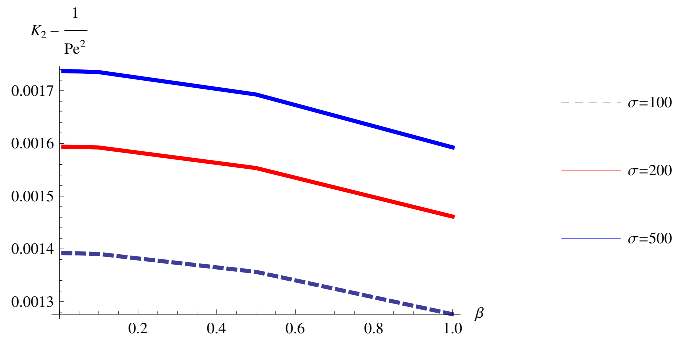

Figure 8. Plots of scale dispersion coefficient $K_2 - Pe^{-2}$ versus β for different values of σ when $M = 1$ and $a = 5$

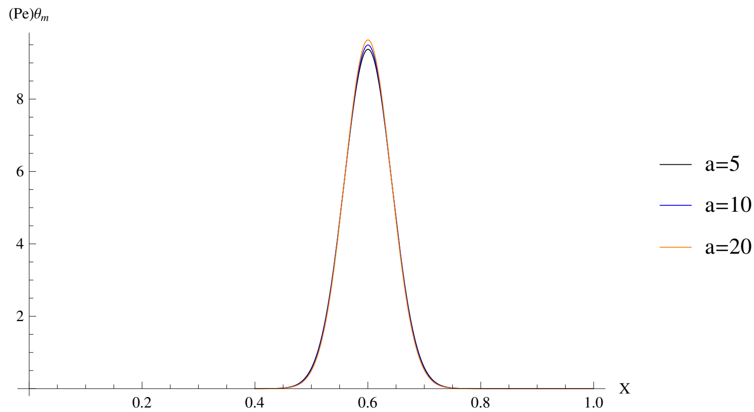

Figure 9. Plots of mean concentration θ_m versus X for different values of a when $M = 1, \sigma = 100, \beta = 10^{-2}$ and $\tau = 0.6$

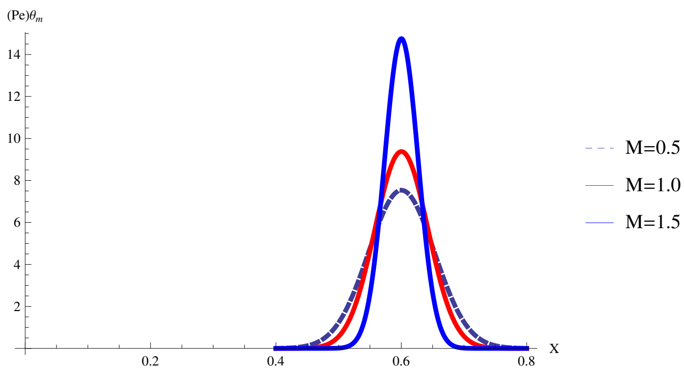

Figure 10.Plots of mean concentration θ_m versus X for different values of M when $a = 5, \sigma = 100, \beta = 10^{-2}$ and $\tau = 0.6$

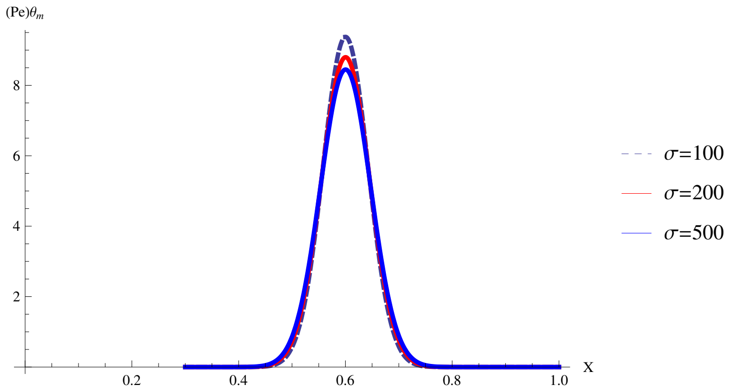

Figure 11. Plots of mean concentration θ_m versus X for different values of σ when $M = 1, a = 5, \beta = 10^{-2}$ and $\tau = 0.6$

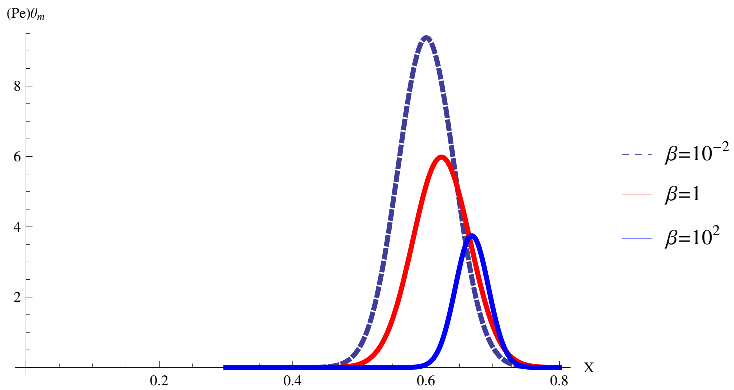

Figure 12. Plots of mean concentration θ_m versus X for different values of β when $M = 1, a = 5, \sigma = 100$ and $\tau = 0.6$

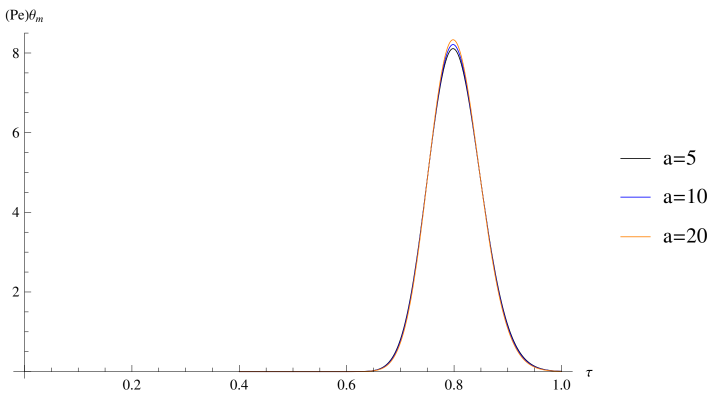

Figure 13.Plots of mean concentration θ_m versus τ for different values of a when $M = 1, \sigma = 100, \beta = 10^{-2}$ and $X = 0.8$

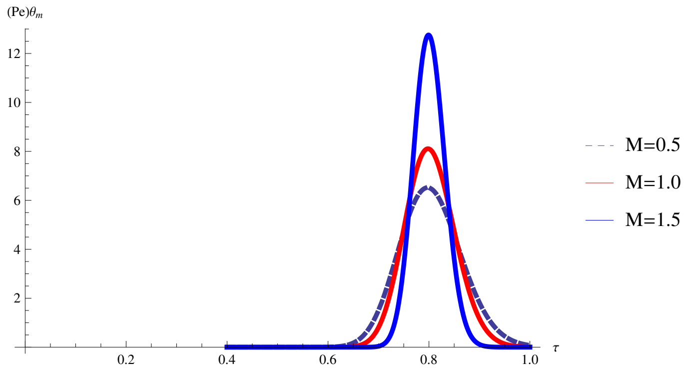

Figure 14.Plots of mean concentration θ_m **versus** τ **for different values of** M **when** $a = 5, \sigma = 100, \beta = 10^{-2}$ **and** $X = 0.8$

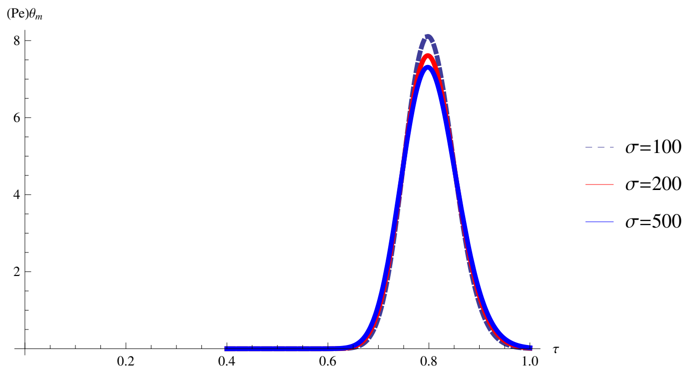

Figure 15.Plots of mean concentration θ_m **versus** τ **for different values of** σ **when** $M = 1, a = 5, \beta = 10^{-2}$ **and** $X = 0.8$

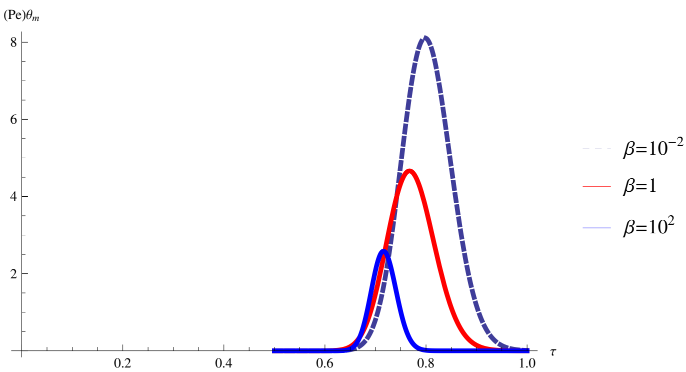

Figure 16. Plots of mean concentration θ_m **versus** τ **for different values of** β **when** $M = 1, a = 5, \sigma = 100$ **and** $X = 0.8$

APPENDIX A

$$m_1 = \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}, \quad Eq.(A.1)$$

$$m_3 = \frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}, \quad Eq.(A.2)$$

$$a_3 = (m_1 + \alpha\sigma)e^{m_1}, \quad Eq.(A.3)$$

$$a_4 = (m_1 - \alpha\sigma)e^{-m_1}, \quad Eq.(A.4)$$

$$a_5 = (m_3 + \alpha\sigma)e^{m_3}, \quad Eq.(A.5)$$

$$a_6 = (m_3 - \alpha\sigma)e^{-m_3}, \quad Eq.(A.6)$$

$$a_7 = \left(\frac{P}{M^2} - \frac{k}{\mu(1+\beta_1)}\frac{\partial p}{\partial x}\right)\alpha\sigma, \quad Eq.(A.7)$$

$$a_8 = m_1^2 e^{m_1}, a_9 = m_1^2 e^{-m_1} \quad Eq.(A.8)$$

$$a_{10} = m_3^2 e^{m_3}, a_{11} = m_3^2 e^{-m_3}, \quad Eq.(A.9)$$

$$C_1 = C_2 = \frac{-a_7a_{10} - a_7a_{11}}{a_5a_8 - a_6a_8 + a_5a_9 - a_6a_9 - a_3a_{10} + a_4a_{10} - a_3a_{11} + a_4a_{11}}, \quad Eq.(A.10)$$

$$C_3 = C_4 = \frac{a_7a_8 + a_7a_9}{a_5a_8 - a_6a_8 + a_5a_9 - a_6a_9 - a_3a_{10} + a_4a_{10} - a_3a_{11} + a_4a_{11}}, \quad Eq.(A.11)$$

$$A_1 = \frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{P}{M^2}, \quad Eq.(A.12)$$

$$A_2 = \frac{C_1 \sinh m_1}{m_1} + \frac{C_3 \sinh m_3}{m_3}, \quad Eq.(A.13)$$

$$A_3 = \frac{C_1 \sinh m_1}{m_1^3} + \frac{C_3 \sinh m_3}{m_3^3}. \quad Eq.(A.14)$$

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