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COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE USING THE PROPERTY (CLRg)

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Abstract The main purpose of this paper is to give common fixed point theorem in intuitionistic fuzzy metric spaces under strict contractive conditions for mappings satisfying (CLRg) property .Our result improves and generalizes the result of Sharma, P and Sharma, S.

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1. Introduction

The concept of fuzzy sets was introduced initially by Zadeh [41] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, Atanassove [5] introduced the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and non-nearness. Coker [8] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [3] proved the well- known fixed point theorems of Banach [6] in the setting of intuitionisitc fuzzy metric spaces. Later on, Turkoglu et al. [38] Proved Jungck's [13] common fixed point theorem in the setting of intuitionisitc fuzzy metric space. Turkoglu et al. [38] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of pant's theorem [23]. Gregori et al. [11], Saadati and Park [29] studied the concept of intuitionistic fuzzy metric space and its applications. Recently,

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many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric space (See [7], [10], [12], [14], [20], [26], [27], [34], [35], [36], [37], [39], [40]).

The study of common fixed points of non compatible mappings is also very interesting. Work along these lines has recently been initiated by pant [24, 25]. Kamran [17] obtained some coincidence and fixed point theorems for hybrid strict contractions.

In 2002, Aamri and El- Moutawakil defined the notion of property (E-A) in metric spaces for self mappings which contained the class of non compatible mappings in metric spaces. Sharma and Bamboria [33] defined a property (S-B) in fuzzy metric spaces for self maps and obtained some common fixed point theorems in IFMS for such mappings under strict contractive conditions. The class of (S-B) maps contains the class of non compatible maps.

Most recently, Sintunavarat and Kumam [34] defined the notion of "common limit in the range" property or CLR property in fuzzy metric spaces. It is observed that the notion of CLR property never requires the condition of the closedness of the subspace while (E-A) and (S-B) property require this condition for the existence of the fixed point.

The object of this paper is proving a common fixed point theorem in IFMS by using the notion of CLR property.

Definition 1.1: **[30]** A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if * is satisfying the following condition;

(i) $*$ is commutative and associative;
(ii) $*$ is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$; (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.2: **[30]** A binary operation $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\diamondsuit$ is satisfying the following conditions;

(i) $\diamondsuit$ is commutative and associative;
(ii) $\diamondsuit$ is continuous
(iii) $a \diamondsuit 0 = a$ for all $a \in [0,1]$;
(iv) $a \diamondsuit b \leq c \diamondsuit d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.3: **[3]** A 5 tuple $(X, M, N, *, \diamondsuit)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm X^2 and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions;

(i) $M(x,y,t) + N(x,y,t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x,y,0) = 0$ for all $x, y \in X$;
(iii) $M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y,t) = M(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ for all $x, y \in z, X$ and $s, t > 0$;
(vi) for all $x, y \in X, M(x,y,.) : [0,\infty) \to$ is left continuous;
(vii) $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x, y \in X$;
(ix) $N(x,y,t) = 0$ for all $x, y \in X$ and $t > 0$ iff $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \diamondsuit N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X, N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous;
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X.
(M, N) is called an intuitionistic fuzzy metric on X.

The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Remark 1.4: **[2]** An intuitionistic fuzzy metric spaces with continous t-norm $*$ and continous t-conorm $\diamondsuit$ defined by $a * a \geq a$ and $(1 - a)\diamondsuit(1 - a) \leq (1 - a)$ for all $a \in [0, 1]$. Then for all $x, y \in X, M(x, y, *)$ is non- decreasing and $N(x, y, \diamondsuit)$ is non- increasing.

Alaca, Turkoglu and Yildiz [3] introduced the following notions;

Definition 1.5: Let $(X, M, N*, \diamondsuit)$ be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.
(b) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if , for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, t) = 1, \lim_{n\to\infty} N(x_n, x, t) = 0$;

Since $*$ and $\diamondsuit$ are continuous, the limit is uniquely determined from (v) and (xi) of definition 3, respectively. An intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 1.6: **[35]** A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to be compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

Definition 1.7: A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to be non-compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ or non existent and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or non existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

In 1998, Jungck and Rhodes [16] introduced the concept of weakly compatible maps as follows;

Definition 1.8: Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Aamri and Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion of E.A. Property and proved common fixed point theorems using this property. Sharma and Bamboria [33] defined the (S-B) property and proved common fixed point theorems in fuzzy metric spaces using this property.

Definition 1.9: A pair of self mappings (S, T) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to satisfy (S-B) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\to\infty} N(Sx_n, Tx_n, t) = 0$.

Example 1.10: Let $X = [0, \infty)$ consider $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t+d(x, y)]$ and $N(x, y, t) = d(x, y)/[t+d(x, y)]$ where d is usual metric. Define $T, S : X \to [0, \infty)$ by $Tx = x/5$ and $Sx = 2x/5$ for all x in X. Consider $x_n = 1/n$. Now, $\lim_{n\to\infty} M(Sx_n, Tx_n, t) = 1$ and $\lim_{n\to\infty} N(Sx_n, Tx_n, t) = 0$. Therefore S and T satisfy property (S-B).

Definition 1.11: **[34]** A pair of self mappings (f, g) of self mappings defined on an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to satisfy the (CLRg) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu$, for some $u \in X$.

Example 1.12: Let $X = [0, \infty)$ consider $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t + d(x, y)]$ and $N(x, y, t) = d(x, y)/[t + d(x, y)]$ where d is usual metric. Define $f, g : X \to [0, \infty)$ by $f(x) = x + 5$ and $g(x) = 6x$ for all x in X. Consider $\{x_n\} = \{1 + 1/n\}$ in X,we have $\lim_{n\to\infty} f(1 + 1/n) = \lim_{n\to\infty} g(6 + 1/n) = 6 = g(1) = \lim_{n\to\infty}(6 + 6/n) = \lim_{n\to\infty} g(1 + 1/n)$, which shows that the pair (f, g) satisfy property (CLRg).

Lemma 1.13: **[2]** Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1), M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.

Sharma and Sharma [common fixed point theorem in intuitionistic fuzzy metric space under (S-B) property, j. of non linear analysis and optimization, vol.3, no.2 (2012), (161-169)] proved the following.

Theorem A: Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space with $a * a \geq a$ and $(1 - a)\diamondsuit(1 - a) \leq (1 - a)$ for all $a \in [0, 1)$, let A, B, S and T be self mappings of X into itself such that,

(A.1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,
(A.2) (A, S) or (B, T) satisfies the property (S-B).
(A.3) there exists a number $k \in (0, 1)$ such that

$$\Big[1+PM(Sx,Ty,kt)\Big]*M(Ax,By,kt) \geq \varnothing\Big[P\Big\{M(Ax,Sx,kt)*M(By,Ty,kt)+M(Ax,Ty,kt)*M(By,Sx,kt)\Big\} +M(Sx,Ty,t)*M(Ax,Sx,t)*M(By,Ty,t)*M(By,Sx,t)*M(Ax,Ty,(2-\alpha)t)\Big]$$

and

$$\Big[1+PN(Sx,Ty,kt)\Big]\diamondsuit N(Ax,By,kt) \leq \Psi\Big[P\Big\{N(Ax,Sx,kt)\diamondsuit N(By,Ty,kt)+N(Ax,Ty,kt)\diamondsuit N(By,Sx,kt)\Big\} +N(Sx,Ty,t)\diamondsuit N(Ax,Sx,t)\diamondsuit N(By,Ty,t)\diamondsuit N(By,Sx,t)\diamondsuit N\Big(Ax,Ty,(2-\alpha)t\Big)\Big]$$

for all $x, y \in X, P \geq 0, \alpha \in (0,2)$ and $t > 0$. Where $\varnothing, \Psi : [0,1] \to [0,1]$ is continuous function such that $\varnothing(S) > S$ and $\Psi(S) < S$ for each $0 < S < 1$ with $M(x,y,t) > 0$.

(A.4) the pairs (A,S) and (B,T) are weakly compatible,
(A.5) one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X. Then A, B, S and T have a unique common fixed point in X.

2. Main Results

Now, we prove our main result which generalizes the theorems A.

Theorem 2: Let A, B, S and T be self maps of a intuitionistic fuzzy metric spaces $(X, M, N, *, \diamondsuit)$ with continuous t-norm $*$ and continuous t-conorm $\diamondsuit$ defined by $a * a \geq a$ and $(1-a)\diamondsuit(1-a) \leq (1-a)$ for all $a \in [0,1]$ satisfying the following conditions;

(2.1) $B(X) \subseteq S(X)$ and the pair (B,T) satisfies property (CLR_T), or $A(X) \subseteq T(X)$ and the pair (A,S) satisfies property (CLR_S)
(2.2) pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible.
(2.3) there exists a number $k \in (0,1)$ such that

$$\Big[1+PM(Sx,Ty,kt)\Big]*M(Ax,By,kt) \geq \varnothing\Big[P\Big\{M(Ax,Sx,kt)*M(By,Ty,kt)+M(Ax,Ty,kt)*M(By,Sx,kt)\Big\} +M(Sx,Ty,t)*M(Ax,Sx,t)*M(By,Ty,t)*M(By,Sx,t)*M\Big(Ax,Ty,(2-\alpha)t\Big)\Big]$$

and

$$\Big[1+PN(Sx,Ty,kt)\Big]\diamondsuit N(Ax,By,kt)\leq \Psi\Big[P\Big\{N(Ax,Sx,kt)\diamondsuit N(By,Ty,kt)+N(Ax,Ty,kt)\diamondsuit N(By,Sx,kt)\Big\} +N(Sx,Ty,t)\diamondsuit(N(Ax,Sx,t)\diamondsuit N(By,Ty,t)\diamondsuit N(By,Sx,t)\diamondsuit N\Big(Ax,Ty,(2-\alpha)t\Big)\Big]$$

for all $x, y \in X, P \geq 0, \alpha \in (0,2)$ and $t > 0$. Where $\varnothing, \Psi : [0,1] \to [0,1]$ is continuous function such that $\varnothing(S) > S$ and $\Psi(S) < S$ for each $0 < S < 1$ with $M(x,y,t) > 0$.

Then A, B, S and T have a unique common fixed point in X.

Proof. Without loss of generality, assume that $B(X) \subseteq S(X)$ and the pair (B,T) satisfies property (CLR_T) then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = Tx$ for some $x \in X$.

Since $B(X) \subseteq S(X)$ there exists a sequence $\{y_n\} \in X$ such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sy_n = Tx$

Now we shall show that $\lim_{n\to\infty} Ay_n = Tx$. Let $Tx = z$. Taking $y = x_n$ and $x = y_n$

From (2.3), for $\alpha = 1 - q, q \in (0,1)$ we have;

$$\Big[1+PM(Sy_n,Tx_n,kt)\Big]*M(Ay_n,Bx_n,kt)\geq\varnothing\Big[P\Big\{M(Ay_n,Sy_n,kt)*M(Bx_n,Tx_n,kt) +M(Ay_n,Tx_n,kt)*M(Bx_n,Sy_n,kt)\Big\} +M(Sy_n,Tx_n,t)*M(Ay_n,Sy_n,t)* M(Bx_n,Tx_n,t)*M(Bx_n,Sy_n,t)*M\Big(Ay_n,Tx_n,(2-\alpha)t\Big)\Big]$$

and

$$\Big[1+PN(Sy_n,Tx_n,kt)\Big]*N(Ay_n,Bx_n,kt)\leq\Psi\Big[P\{N(Ay_n,Sy_n,kt)\diamondsuit N(Bx_n,Tx_n,kt) +N(Ay_n,Tx_n,kt)\diamondsuit N(Bx_n,Sy_n,kt)\} +N(Sy_n,Tx_n,t)\diamondsuit N(Ay_n,Sy_n,t)\diamondsuit N(Bx_n,Tx_n,t)\diamondsuit N(Bx_n,Sy_n,t)\diamondsuit N\Big(Ay_n,Tx_n,(2-\alpha)t\Big)\Big]$$

$$M(Ay_n,Bx_n,kt)+P\Big[M(Sy_n,Tx_n,kt)*M(Ay_n,Bx_n,kt)\Big]\geq\varnothing\Big[P\{M(Ay_n,Sy_n,kt)*M(Bx_n,Tx_n,kt) +M(Ay_n,Tx_n,kt)*M(Bx_n,Sy_n,kt)\} +M(Sy_n,Tx_n,t)*M(Ay_n,Sy_n,t)*M(Bx_n,Tx_n,t)* M(Bx_n,Sy_n,t)*M\Big(Ay_n,Tx_n,(1+q)t\Big)\Big]$$

and

$$N(Ay_n,Bx_n,kt)+P\Big[N(Sy_n,Tx_n,kt)\diamondsuit N(Ay_n,Bx_n,kt)\Big]\leq\Psi\Big[P\{N(Ay_n,Sy_n,kt)\diamondsuit N(Bx_n,Tx_n,kt) +N(Ay_n,Tx_n,kt)\diamondsuit N(Bx_n,Sy_n,kt)\} +N(Sy_n,Tx_n,t)\diamondsuit N(Ay_n,Sy_n,t)\diamondsuit N(Bx_n,Tx_n,t)\diamondsuit N(Bx_n,Sy_n,t)\diamondsuit N\Big(Ay_n,Tx_n,(1+q)t\Big)\Big]$$

$$M(Ay_n,Bx_n,kt)+P\Big[M(Bx_n,Tx_n,kt)*M(Ay_n,Bx_n,kt)\Big]\geq\varnothing\Big[P\{M(Ay_n,Bx_n,kt)*M(Bx_n,Tx_n,kt) +M(Ay_n,Tx_n,kt)*M(Bx_n,Bx_n,kt)\} +M(Bx_n,Tx_n,t)*M(Ay_n,Bx_n,t)*M(Bx_n,Tx_n,t)* M(Bx_n,Bx_n,t)*M(Ay_n,Bx_n,t)*M(Bx_n,Tx_n,qt)\Big]$$

and

$$N(Ay_n,Bx_n,kt)+P\Big[N(Bx_n,Tx_n,kt)\diamondsuit N(Ay_n,Bx_n,kt)\Big]\leq\Psi\Big[P\{N(Ay_n,Bx_n,kt)\diamondsuit N(Bx_n,Tx_n,kt) +N(Ay_n,Tx_n,kt)\diamondsuit N(Bx_n,Bx_n,kt)\} +N(Bx_n,Tx_n,t)\diamondsuit N(Ay_n,Bx_n,t)\diamondsuit N(Bx_n,Tx_n,t)\diamondsuit N(Bx_n,Bx_n,t)\diamondsuit N(Ay_n,Bx_n,t)\diamondsuit N(Bx_n,Tx_n,qt)\Big]$$

Thus it follows that,

$$M(Ay_n, Bx_n, kt) \geq \varnothing\big[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, qt)\big]$$

and

$$N(Ay_n, Bx_n, kt) \leq \Psi\big[N(Bx_n, Tx_n, t)\diamondsuit N(Ay_n, Bx_n, t)\diamondsuit N(Bx_n, Tx_n, qt)\big]$$

Since the t-norm $*$ and t-conorm $\diamondsuit$ is continuous and $M(x, y, .)$ and $N(x, y, .)$ is continuous, letting $q \to 1$ we have,

$$M(Ay_n, Bx_n, kt) \geq \varnothing\big[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t)\big]$$

and

$$N(Ay_n, Bx_n, kt) \leq \Psi\big[N(Bx_n, Tx_n, t)\diamondsuit N(Ay_n, Bx_n, t)\big]$$

It follows that

$$\lim_{n\to\infty} M(Ay_n, Bx_n, kt) \geq \varnothing\big[\lim_{n\to\infty} M(Ay_n, Bx_n, t), M(\lim_{n\to\infty} Ay_n, z, kt)\big]$$
$$> M(\lim_{n\to\infty} Ay_n, z, t)$$

and

$$\lim_{n\to\infty} N(Ay_n, Bx_n, kt) \leq \Psi\big[\lim_{n\to\infty} N(Ay_n, Bx_n, t), N(\lim_{n\to\infty} Ay_n, z, kt)\big]$$
$$< N(\lim_{n\to\infty} Ay_n, z, t)$$

and we deduce that $\lim_{n\to\infty} Ay_n = z$.

Subsequently, we have $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ay_n = Tx$.

Now we shall show that $Bx = Tx$.

Similarly taking $x = y_n$ and $y = x$ in (2.3), we have;

$$\big[1+PM(Sy_n,Tx,kt)\big]*M(Ay_n,Bx,kt)\geq\varnothing\big[P\{M(Ay_n,Sy_n,kt)*M(Bx,Tx,kt)+M(Ay_n,Tx,kt)*$$
$$M(Bx,Sy_n,kt)\}+M(Sy_n,Tx,t)*M(Ay_n,Sy_n,t)*$$
$$M(Bx,Tx,t)*M(Bx,Sy_n,t)*M\big(Ay_n,Tx,(2-\alpha)t\big)\big]$$

and

$$\big[1+PN(Sy_n,Tx,kt)\big]*N(Ay_n,Bx,kt)\leq\Psi\big[P\{N(Ay_n,Sy_n,kt)\diamondsuit N(Bx,Tx,kt)+N(Ay_n,Tx,kt)\diamondsuit$$
$$N(Bx,Sy_n,kt)\}+N(Sy_n,Tx,t)\diamondsuit N(Ay_n,Sy_n,t)\diamondsuit N(Bx,Tx,t)\diamondsuit$$
$$N(Bx,Sy_n,t)\diamondsuit N\big(Ay_n,Tx,(2-\alpha)t\big)\big]$$

Taking $\lim_{n\to\infty}$ we have;

$$\big[1+PM(Tx,Tx,kt)]*M(Tx,Bx,kt)\geq\varnothing\big[P\{M(Tx,Tx,kt)*M(Bx,Tx,kt)+M(Tx,Tx,kt)*$$
$$M(Bx,Tx,kt)\}+M(Tx,Tx,t)*M(Tx,Tx,t)*$$
$$M(Bx,Tx,t)*M(Bx,Tx,t)*M\big(Tx,Tx,(2-\alpha)t\big)\big]$$

and

$$\big[1+PN(Tx,Tx,kt)\big]*N(Tx,Bx,kt)\leq\Psi\big[P\{N(Tx,Tx,kt)\diamondsuit N(Bx,Tx,kt)+N(Tx,Tx,kt)\diamondsuit$$
$$N(Bx,Tx,kt)\}+N(Tx,Tx,t)\diamondsuit N(Tx,Tx,t)\diamondsuit$$
$$N(Bx,Tx,t)\diamondsuit N(Bx,Tx,t)\diamondsuit N\big(Tx,Tx,(2-\alpha)t\big)\big]$$

It follows that (for $\alpha = 1$)

$$M(Tx, Bx, kt) \geq \varnothing\big[P\{1 * M(Bx, Tx, kt) + 1 * M(Bx, Tx, kt)\}$$
$$+ 1 * 1 * M(Bx, Tx, t) * M(Bx, Tx, t) * 1\big]$$

and

$$N(Tx, Bx, kt) \leq \Psi\big[P\{0\diamondsuit N(Bx, Tx, kt) + 0\diamondsuit N(Bx, Tx, kt)\}$$
$$+ 0\diamondsuit 0\diamondsuit N(Bx, Tx, t)\diamondsuit N(Bx, Tx, t)\diamondsuit 0\big]$$

Thus it follows that,

$$M(Tx,Bx,kt) > M(Bx,Tx,t) \text{ and } N(Tx,Bx,kt) < N(Bx,Tx,t)$$

By lemma 1.13 we have $Bx = Tx$.

Let $Bx = Tx = z$. Since the pair (B,T) is weak compatible, it follows that $Bz = Tz$. $\cdots(1)$

Also since $B(X) \subseteq S(X)$, there exists some y in X such that $Bx = Sy(= z)$.

We next show that $Sy = Ay(= z)$.

Taking $y = x_n, x = y$ in (2.3), we have;

$$\begin{aligned}\Big[1+PM(Sy,Tx_n,kt)\Big]*M(Ay,Bx_n,kt)\geq\varnothing\Big[&P\{M(Ay,Sy,kt)*M(Bx_n,Tx_n,kt)+M(Ay,Tx_n,kt)*\\&M(Bx_n,Sy,kt)\}+M(Sy,Tx_n,t)*M(Ay,Sy,t)*\\&M(Bx_n,Tx_n,t)*M(Bx_n,Sy,t)*M\Big(Ay,Tx_n,(2-\alpha)t\Big)\Big]\end{aligned}$$

and

$$\begin{aligned}\Big[1+PN(Sy,Tx_n,kt)\Big]*N(Ay,Bx_n,kt)\leq\Psi\Big[&P\{N(Ay,Sy,kt)\diamondsuit N(Bx_n,Tx_n,kt)+N(Ay,Tx_n,kt)\diamondsuit\\&N(Bx_n,Sy,kt)\}+N(Sy,Tx_n,t)\diamondsuit N(Ay,Sy,t)\diamondsuit\\&N(Bx_n,Tx_n,t)\diamondsuit N(Bx_n,Sy,t)\diamondsuit N\Big(Ay,Tx_n,(2-\alpha)t\Big)\Big]\end{aligned}$$

Taking $\lim_{n\to\infty}$ we have;

$$\begin{aligned}\Big[1+PM(Sy,Sy,kt)\Big]*M(Ay,Sy,kt)\geq\varnothing\Big[&P\{M(Ay,Sy,kt)*M(Tx,Tx,kt)+M(Ay,Sy,kt)*\\&M(Sy,Sy,kt)\}+M(Sy,Sy,t)*M(Ay,Sy,t)*\\&M(Tx,Tx,t)*M(Sy,Sy,t)*M\Big(Ay,Sy,(2-\alpha)t\Big)\Big]\end{aligned}$$

and

$$\begin{aligned}\Big[1+PN(Sy,Sy,kt)]*N(Ay,Sy,kt)\leq\Psi\Big[&P\{N(Ay,Sy,kt)\diamondsuit N(Tx,Tx,kt)+N(Ay,Sy,kt)\diamondsuit\\&N(Sy,Sy,kt)\}+N(Sy,Sy,t)\diamondsuit N(Ay,Sy,t)\diamondsuit\\&N(Tx,Tx,t)\diamondsuit N(Sy,Sy,t)\diamondsuit N\Big(Ay,Sy,(2-\alpha)t\Big)\Big]\end{aligned}$$

It follows that

$$\begin{aligned}M(Ay,Sy,kt) \geq \varnothing\big[&P\{M(Ay,Sy,kt)*1+M(Ay,Sy,kt)*1\}\\&+1*M(Ay,Sy,t)*1*1*M\big(Ay,Sy,(2-\alpha)t\big)\big]\end{aligned}$$

and

$$\begin{aligned}N(Ay,Sy,kt) \leq \Psi\big[&P\{N(Ay,Sy,kt)\diamondsuit 0+N(Ay,Sy,kt)\diamondsuit 0\}\\&+0\diamondsuit N(Ay,Sy,t)\diamondsuit 0\diamondsuit 0\diamondsuit N\big(Ay,Sy,(2-\alpha)t\big)\big]\end{aligned}$$

Thus it follows that (for $\alpha = 1$)

$$M(Ay,Sy,kt) > M(Ay,Sy,t) \text{ and } N(Ay,Sy,kt) < N(Ay,Sy,t)$$

By lemma 1.13 finally we have $Sy = Ay(= z)$.

But the pair (A, S) is weakly compatible, it follows that $Az = Sz$. $\cdots(2)$

Next, we claim that $Az = Bz$.

Taking $x = z, y = z$ in (2.3) and applying (1) and (2) we have;

$$\begin{aligned}\left[1+PM(Sz,Tz,kt)\right]*M(Az,Bz,kt)\geq\varnothing\Big[P\{M(Az,Sz,kt)\Big]*M(Bz,Tz,kt)+M(Az,Tz,kt)*\\ M(Bz,Sz,kt)\}+M(Sz,Tz,t)*M(Az,Sz,t)*\\ M(Bz,Tz,t)*M(Bz,Sz,t)*M\Big(Az,Tz,(2-\alpha)t\Big)\Big]\end{aligned}$$

and

$$\begin{aligned}\Big[1+PN(Sz,Tz,kt)\Big]\diamondsuit N(Az,Bz,kt)\leq\Psi\Big[P\{N(Az,Sz,kt)\diamondsuit N(Bz,Tz,kt)+N(Az,Tz,kt)\diamondsuit\\ N(Bz,Sz,kt)\}+N(Sz,Tz,t)\diamondsuit(N(Az,Sz,t)\diamondsuit\\ N(Bz,Tz,t)\diamondsuit N(Bz,Sz,t)\diamondsuit N\Big(Az,Tz,(2-\alpha)t\Big)\Big]\end{aligned}$$

$$\begin{aligned}\Big[1+PM(Az,Bz,kt)\Big]*M(Az,Bz,kt)\geq\varnothing\Big[P\{M(Az,Az,kt)\Big]*M(Bz,Bz,kt)+M(Az,Bz,kt)*\\ M(Bz,Az,kt)\}+M(Az,Bz,t)*M(Az,Az,t)*\\ M(Bz,Bz,t)*M(Bz,Az,t)*M\Big(Az,Bz,(2-\alpha)t\Big)\Big]\end{aligned}$$

and

$$\begin{aligned}\Big[1+PN(Az,Bz,kt)\Big]\diamondsuit N(Az,Bz,kt)\leq\Psi\Big[P\{N(Az,Az,kt)\diamondsuit N(Bz,Bz,kt)+N(Az,Bz,kt)\diamondsuit\\ N(Bz,Az,kt)\}+N(Az,Bz,t)\diamondsuit(N(Az,Az,t)\diamondsuit\\ N(Bz,Bz,t)\diamondsuit N(Bz,Az,t)\diamondsuit N\Big(Az,Bz,(2-\alpha)t\Big)\Big]\end{aligned}$$

This implies that

$$\begin{aligned}M(Az,Bz,kt)\geq\varnothing\big[P\{1*1+M(Az,Bz,kt)*M(Bz,Az,kt)\}\\ +M(Az,Bz,t)*1*1*M(Bz,Az,t)*M\big(Az,Bz,(2-\alpha)t\big)\big]\end{aligned}$$

and

$$\begin{aligned}N(Az,Bz,kt)\leq\Psi\big[P\{0\diamondsuit 0+N(Az,Bz,kt)\diamondsuit N(Bz,Az,kt)\}\\ +N(Az,Bz,t)\diamondsuit 0\diamondsuit 0\diamondsuit N(Bz,Az,t)\diamondsuit N\big(Az,Bz,(2-\alpha)t\big)\big]\end{aligned}$$

For $\alpha = 1$, we have;

$$M(Az,Bz,kt) > M(Bz,Az,t) \text{ and } N(Az,Bz,kt) < N(Az,Bz,t)$$

By lemma 1.13 we have $Az = Bz$. $\cdots(3)$

Hence by (1), (2) and (3) we have $Az = Bz = Sz = Tz$. $\cdots(4)$

We now show that $z = Az$. Taking $x = z, y = x$ in (2.3) and applying (4) and previous known result like $Bx = Tx = z$ we have;

$$\begin{aligned}\left[1+PM(Sz,Tx,kt)\right]*M(Az,Bx,kt)\geq\varnothing\Big[P\{M(Az,Sz,kt)\Big]*M(Bx,Tx,kt)+M(Az,Tx,kt)*\\ M(Bx,Sz,kt)\}+M(Sz,Tx,t)*M(Az,Sz,t)*\\ M(Bx,Tx,t)*M(Bx,Sz,t)*M\Big(Az,Tx,(2-\alpha)t\Big)\Big]\end{aligned}$$

and

$$\Big[1+PN(Sz,Tx,kt)\Big]?N(Az,Bx,kt)\le\Psi\Big[P\{N(Az,Sz,kt)\Diamond N(Bx,Tx,kt)+N(Az,Tx,kt)\Diamond N(Bx,Sz,kt)\}+N(Sz,Tx,t)\Diamond(N(Az,Sz,t)\Diamond N(Bx,Tx,t)\Diamond N(Bx,Sz,t)\Diamond N\Big(Az,Tx,(2-\alpha)t\Big)\Big]$$

$$\Big[1+PM(Az,z,kt)\Big]*M(Az,z,kt)\ge\varnothing\Big[P\{M(Az,Az,kt)*M(Bx,Bx,kt)+M(Az,z,kt)*M(z,Az,kt)\}+M(Az,z,t)*M(Az,Az,t)*M(Bx,Bx,t)*M(z,Az,t)*M\alpha(Az,z,(2-\alpha)t\Big)\Big]$$

and

$$\Big[1+PN(Az,z,kt)\Big]\Diamond N(Az,z,kt)\le\Psi\Big[P\{N(Az,Az,kt)\Diamond N(Bx,Bx,kt)+N(Az,z,kt)\Diamond N(z,Az,kt)\}+N(Az,z,t)\Diamond(N(Az,Az,t)\Diamond N(Bx,Bx,t)\Diamond N(z,Az,t)\Diamond N\Big(Az,z,(2-\alpha)t\Big)\Big]$$

$$M(Az,z,kt)\ge\varnothing\big[P\{1*1+M(Az,z,kt)*M(z,Az,kt)\}+M(Az,z,t)*1*1*M(z,Az,t)*M\big(Az,z,(2-\alpha)t\big)\big]$$

and

$$N(Az,z,kt)\le\Psi\big[P\{0\Diamond 0+N(Az,z,kt)\Diamond N(z,Az,kt)\}+N(Az,z,t)\Diamond 0\Diamond 0\Diamond N(z,Az,t)\Diamond N\big(Az,z,(2-\alpha)t\big)\big]$$

For $\alpha=1$, we have;

$$M(Az,z,kt)>M(Az,z,t) \text{ and } N(Az,z,kt)<N(Az,z,t)$$

By lemma 1.13 we have; $Az=z$. $\cdots(5)$

Hence by (4) and (5) we have $Az=Bz=Sz=Tz=z$, that is z is the common fixed point of the maps A,B,S and T. Uniqueness follows easily.

The proof is similar when we assume $A(X)\subseteq T(X)$ and the pair (A,S) satisfies property (CLR_S).

If we put $P=0$, we get the following result.

Corollary 2.1: Let A,B,S and T be self maps of a intuitionistic fuzzy metric spaces $(X,M,N,*,\Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $a*a\ge a$ and $(1-a)\Diamond(1-a)\le(1-a)$ for all $a\in[0,1]$ satisfying the following conditions;

(2.1) $B(X)\subseteq S(X)$ and the pair (B,T) satisfies property (CLR_T), or $A(X)\subseteq T(X)$ and the pair (A,S) satisfies property (CLR_S)
(2.2) pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible.
(2.3) there exists a number $k\in(0,1)$ such that

$$M(Ax,By,kt)\ge\varnothing\big[M(Sx,Ty,t)*M(Ax,Sx,t)*M(By,Ty,t)*M(By,Sx,t)\}*M\big(Ax,Ty,(2-\alpha)t\big)\big]$$

and

$$N(Ax, By, kt) \leq \Psi\big[N(Sx, Ty, t)\Diamond N(Ax, Sx, t)\Diamond N(By, Ty, t)\Diamond N(By, Sx, t)\}\Diamond \\ N\big(Ax, Ty, (2-\alpha)t\big)\big]$$

for all $x, y \in X, P \geq 0, \alpha \in (0, 2)$ and $t > 0$. Where $\varnothing, \Psi : [0, 1] \to [0, 1]$ is continuous function such that $\varnothing(S) > S$ and $\Psi(S) < S$ for each $0 < S < 1$ with $M(x, y, t) > 0$. Then A, B, S and T have a unique common fixed point in X.

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