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Some Common Coupled Fixed Points of Mappings Satisfying Contractive Conditions with Rational Expressions in Complex Valued G_b-Metric Spaces

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Abstract In this paper, the notion of complex valued G_b-metric space is introduced by using the setting of G_b-metric space and complex valued metric space. The existence and uniqueness of common coupled fixed points for mappings involving rational expressions are proved in the framework of a complex valued G_b-metric space. Also, the illustrative example is given in the support of proved theorem.

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1. Introduction

Fixed point theory is a very crucial and versatile tool in many fields such as nonlinear functional theory, game theory, control theory, dynamics and economic theory. Fixed point theorems are used to investigate the existence and uniqueness of solution when solving various mathematical equations. One of the most fundamental and useful fixed point theorems is the Banach contraction principle. It is extensively applied in analysis and applied mathematics. The fantastic significance of Banach's principle relates to the possibility of obtaining the fixed point. Due to its simplicity and importance, this principle has been generalized by many authors in different directions, such as contractive conditions including product and rational expressions, asymptotic contractions, contractions of Meir-Keeler type, weak contractions, etc.
On the other hand, since metric fixed point theory is a fundamental construction of fixed point theory many authors have generalized and have established the notion of a metric spaces in the recent past such as rectangular metric spaces, semi-metric spaces, quasi-metric spaces, quasi-semi metric spaces, pseudo-metric spaces, 2-metric spaces, D-metric

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spaces, G-metric spaces, K-metric spaces, b metric spaces, G_b-metric spaces and etc.
In 1989, Bakhtin [5] introduced the concept of $b-$metric space as a generalization of metric space. After that, several interesting results for the existence of fixed point for single-valued and multivalued operators in $b-$metric spaces have been obtained.
Azam et al.[2] have defined the complex valued metric spaces and also have obtained some fixed point results for a pair of mappings for contraction condition satisfying a rational expressions which are not meaningful in cone metric spaces in 2011. Therefore, many results of analysis can not be generalized to cone metric spaces. Later, one can study progresses of a host of results of analysis involving divisions in the framework of complex valued metric spaces.
Recently, the concept of complex valued $b-$metric spaces was introduced by Rao et al.[16], which are more general than the well-known complex valued metric spaces. Then, Mukheimer [11] established the existence and uniqueness of common fixed point for two self-mappings on complex valued $b-$metric spaces in 2014.
Mustafa and Sims [14] generalized the notion of a metric space. Afterwards, Mustafa et al.[12] obtained some fixed point results for mappings satisfying different contractive conditions. Then, Abbas and Rhoades initiated the study of common fixed point theory in $G-$metric spaces. Recently, Aghajani et al.[3] extended the notion of $G-$metric space to the concept of G_b-metric space. They also proved a common fixed point theorem for six mappings satisfying weakly compatible condition in complete partially ordered G_b-metric spaces.
In 2006, Bhaskar and Lakshmikantham [6] introduced the concept of coupled fixed points for a given partially ordered set X and proved coupled fixed point theorems for mappings which satisfy the mixed monotone property. Since then, Kutbi et al.[10] studied the notion of common coupled fixed points for a pair of mappings on a complex valued metric space along with generalized contraction involving rational expressions. In 2014, Sedghi et al.[19] proved a coupled coincidence fixed point theorem in G_b-metric space.

The aim of this paper is to define a new space called complex valued G_b-metric space by getting together structure of complex valued metric space and circumstances of G_b-metric space. Also, we prove some coupled common fixed point theorem with contractive conditions involving rational expressions in the complex valued G_b-metric space. Also, the example is presented to demonstrate that the main theorem is remarkable.

2. Preliminaries

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\prec$, $\precsim$ on $\mathbb{C}$ as follows:

(i) $z_1 \prec z_2$ if and only if $\text{Re}(z_1) < \text{Re}(z_2)$, $\text{Im}(z_1) < \text{Im}(z_2)$,
(ii) $z_1 \precsim z_2$ if and only if $\text{Re}(z_1) \le \text{Re}(z_2)$, $\text{Im}(z_1) \le \text{Im}(z_2)$.

It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:
(i) $\text{Re}\,(z_1) = \text{Re}\,(z_2)$, $\text{Im}\,(z_1) < \text{Im}\,(z_2)$,
(ii) $\text{Re}\,(z_1) < \text{Re}\,(z_2)$, $\text{Im}\,(z_1) = \text{Im}\,(z_2)$,
(iii) $\text{Re}\,(z_1) < \text{Re}\,(z_2)$, $\text{Im}\,(z_1) < \text{Im}\,(z_2)$,
(iv) $\text{Re}\,(z_1) = \text{Re}\,(z_2)$, $\text{Im}\,(z_1) = \text{Im}\,(z_2)$.

In particular; we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii) and (iii) is satisfied and we write $z_1 \prec z_2$ is only (iii) is satisfied. Note that

$$0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|,$$

$$z_1 \precsim z_2,\ z_2 \prec z_3 \ \Rightarrow \ z_1 \prec z_3.$$

Definition 1. [24] Let $z_1, z_2 \in \mathbb{C}$ and the 'max' function for the partial order relation $\precsim$ is defined on $\mathbb{C}$ by:
(i) $max\{z_1, z_2\} = z_2 \Leftrightarrow z_1 \precsim z_2$;
(ii) $z_1 \precsim max\{z_2, z_3\} \Rightarrow z_1 \precsim z_2$ or $z_1 \precsim z_3$;
(iii) $max\{z_1, z_2\} = z_2 \Leftrightarrow z_1 \precsim z_2$ or $|z_1| \leq |z_2|$.

Lemma 1. [24] Let $z_1, z_2, z_3, ... \in \mathbb{C}$ and the partial order relation $\precsim$ is defined on $\mathbb{C}$. Then the following statements are obvious:
(i) If $z_1 \precsim max\{z_2, z_3\}$, then $z_1 \precsim z_2$ if $z_3 \precsim z_2$;
(ii) If $z_1 \precsim max\{z_2, z_3, z_4\}$ then $z_1 \precsim max\{z_2, z_3, z_4, z_5\}$, if $max\{z_3, z_4\} \precsim z_2$;
(iii) If $z_1 \precsim max\{z_2, z_3, z_4, z_5\}$, then $z_1 \precsim z_2$ if $max\{z_3, z_4, z_5\} \precsim z_2$ and so on.

Now, we will give the definition of G_b-metric space.

Definition 2. [3] Let X be a nonempty set and let $s \geq 1$ be a given real number. Suppose that a mapping $G : X \times X \times X \to \mathbb{R}^+$ satisfies:
(G_b1) $G(x, y, z) = 0$; if $x = y = z$;
(G_b2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$;
(G_b3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$;
(G_b4) $G(x, y, z) = G(p\{x, y, z\})$ where p is a permutation of x, y, z(symmetry);
(G_b5) $G(x, y, z) \leq s[G(x, a, a) + G(a, y, z)]$ for all $x, y, z, a \in X$(rectangle inequality).

Then G is called a generalized $b-$metric and pair (X, G) is called a generalized $b-$metric space or G_b-metric space.

Example 1.[3] Let (X, G) be a $G-$metric space, and $G_*(x, y, z) = G^p(x, y, z)$, where $p > 1$ is a real number. Note that G_* is a G_b-metric with $s = 2^{p-1}$. (X, G_*) is not necessarily a $G-$metric space.

Consistent with [19], some basic notations related with couple fixed point theory needed in this paper are recalled as follows.

Definition 3. Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = gx \quad and \quad F(y, x) = gy.$$

Definition 4. Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if

$$F(x, y) = x \quad and \quad F(y, x) = y.$$

Definition 5. Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a common coupled fixed point of mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = gx = x \quad and \quad F(y, x) = gy = y.$$

Definition 6. Let X be a nonempty set. Then we say that the mappings $F : X \times X \to X$ and $g : X \to X$ are commutative if

$$gF(x,y) = F(gx,gy) \quad \textit{and} \quad gF(y,x) = F(gy,gx).$$

3. Complex Valued G_b-Metric Topology

In this section, we introduce complex valued G_b-metric space, investigate the topological structure and give some definitions, lemmas and example. Throughout the paper, '$\prec$' is taken the partial order relation on $\mathbb{C}$.

Definition 7. Let X be a nonempty set and let $s \geq 1$ be a given real number. Suppose that a mapping $G_{b_c} : X \times X \times X \to \mathbb{C}$ satisfies:
$G_{b_c}1$. $G(x,y,z) = 0$; if $x = y = z$;
$G_{b_c}2$. $0 \prec G(x,x,y)$ for all $x, y \in X$ with $x \neq y$;
$G_{b_c}3$. $G(x,x,y) \precsim G(x,y,z)$ for all $x, y, z \in X$ with $y \neq z$;
$G_{b_c}4$. $G(x,y,z) = G(p\{x,y,z\})$ where p is a permutation of x, y, z(symmetry);
$G_{b_c}5$. $G(x,y,z) \precsim s[G(x,a,a) + G(a,y,z)]$ for all $x, y, z, a \in X$(rectangle inequality).

Then G_{b_c} is called a complex valued G_b-metric on X, the pair (X, G_{b_c}) is called a complex valued G_b-metric space.

Example 2. Let $X = \mathbb{R}$. Define $G : X \times X \times X \to \mathbb{C}$ by

$$G_{b_c}(x,y,z) = \frac{1}{9}\left[(|x-y| + |y-z| + |x-z|)^2 + i(|x-y| + |y-z| + |x-z|)^2\right]$$

for all $x, y, z \in X$. Then (X, G_{b_c}) is a complex valued G_b-metric space with $s = 2$.

Definition 8. A complex valued G_b-metric G_{b_c} is said to be symmetric if $G_{b_c}(x,y,y) = G_{b_c}(y,x,x)$ for all $x, y \in X$.

Let (X, G_{b_c}) be a complex valued G_b-metric space. Then, for $x_0 \in X, 0 \prec r \in \mathbb{C}$, the $G_{b_c}-$ball with center x_0 and radius r is

$$B_{G_b}(x_0, r) = \{y \in X |\ G_{b_c}(x_0,y,y) \prec r\}.$$

Now, we will give an example of $G_{b_c}-$ball.

Let $X = \mathbb{R}$ and consider the complex valued G_b-metric G_{b_c} defined by

$$G_{b_c}(x,y,z) = \frac{1}{9}\left[(|x-y| + |y-z| + |x-z|)^2 + i(|x-y| + |y-z| + |x-z|)^2\right]$$

for all $x, y, z \in X$. Then

$$\begin{aligned} B_{G_b}(1, 4+i4) &= \{y \in X : G_{b_c}(1,y,y) \prec 4+i4\} \\ &= \left\{y \in X : \frac{1}{9}\left[(|y-1| + |y-1|)^2 + i(|y-1| + |y-1|)^2\right] \prec 4+i4\right\} \\ &= \left\{y \in X : \left(|y-1|^2 + i|y-1|^2\right) \prec 9+i9\right\} \\ &= \left\{y \in X : |y-1|^2 \prec 9\right\} = (-2, 4). \end{aligned}$$

A point $x \in X$ is called interior point of a set $A \subseteq X$, whenever there exists $0 \prec r \in \mathbb{C}$ such that

$$B_{G_b}(x,r) = \{y \in X \mid G_{b_c}(x,y,y) \prec r\} \subseteq A.$$

A point $x \in X$ is called limit point of a set A whenever there exists $0 \prec r \in \mathbb{C}$ such that

$$B_{G_b}(x,r) \cap (A - X) \neq \emptyset.$$

A is open whenever each element of A is an interior point of A. A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

Now we present some propositions in complex valued G_b-metric space which are needed to explain the topological structure of the space.

Proposition 1. For each $z_1, z_2 \in \mathbb{C}$, there exists $z \in \mathbb{C}$ such that $z \prec z_1$ and $z \prec z_2$.

Proof. Let us suppose that $z_1, z_2 \in \mathbb{C}$. Since $z_1 \in \mathbb{C}$, then $z_1 = x_1 + iy_1$, for $x_1, y_1 \in \mathbb{R}$. Similarly, because $z_2 \in \mathbb{C}$, then $z_2 = x_2 + iy_2$, for $x_2, y_2 \in \mathbb{R}$. We choose that $x_3 = min\{x_1, x_2\}$ and $y_3 = min\{y_1, y_2\}$. Hence, we have that $x_3 < x_1$ and $x_3 < x_2$. Similarly, we get that $y_3 < y_1$ and $y_3 < y_2$. This implies that $z \prec z_1$ and $z \prec z_2$. As we can write that $z = x_3 + iy_3$, then $z \in \mathbb{C}$. So, the desired result is obtained. ∎

Proposition 2. Let X be a complex valued G_b-metric space, then for each $x, y, z, a \in X$ it follows that:

i. If $G_{b_c}(x,y,z) = 0$ then $x = y = z$;
ii. $G_{b_c}(x,y,z) \precsim s\left[G_{b_c}(x,x,y) + G_{b_c}(x,x,z)\right]$;
iii. $G_{b_c}(x,y,y) \precsim 2sG_{b_c}(y,x,x)$;
iv. $G_{b_c}(x,y,z) \precsim s\left[G_{b_c}(x,a,z) + G_{b_c}(a,y,z)\right]$.

Proof. i. Let $G_{b_c}(x,y,z) = 0$. Suppose that $x \neq y \neq z$. Then from the property of $G_{b_c}1$, we have that $G_{b_c}(x,y,z) \neq 0$, which is a contradiction. So, $x = y = z$.

ii. From the properties $G_{b_c}4$ and $G_{b_c}5$, we have that

$$G_{b_c}(x,y,z) = G_{b_c}(z,y,x) \precsim s\left[G_{b_c}(z,a,a) + G_{b_c}(a,y,x)\right].$$

Then, by taking $a = x$, the desired result is obtained.

iii. By using $G_{b_c}4$ and $G_{b_c}5$, we get that

$$G_{b_c}(x,x,y) = G_{b_c}(x,y,x) \precsim s\left[G_{b_c}(x,a,a) + G_{b_c}(a,y,x)\right].$$

When we replace a with y, this proposition is easily verified.

iv. Combining $G_{b_c}3$ and $G_{b_c}5$ with $a \neq z$, we have
$G_{b_c}(x,y,z) \precsim s\left[G_{b_c}(x,a,a) + G_{b_c}(a,y,z)\right] \precsim s\left[G_{b_c}(x,a,z) + G_{b_c}(a,y,z)\right].$ ∎

Proposition 3. Let (X, G_{b_c}) be complex valued G_b-metric space, then for any $x_0 \in X$ and $0 \prec r \in \mathbb{C}$, we have

i) If $G_{b_c}(x_0,x,y) \prec r$, then $x, y \in B_{G_b}(x_0,r)$;
ii. If $y \in B_{G_b}(x_0,r)$, then there exists a $0 \prec \delta \in \mathbb{C}$ such that $B_{G_b}(y,\delta) \subseteq B_{G_b}(x_0,r)$.

Proof. i. Let us suppose that $G_{b_c}(x_0, x, y) \prec r$ for any $x_0 \in X$ and $0 \prec r$. By the properties $G_{b_c}3$ and $G_{b_c}4$, for $x_0 \neq x \neq y$

$$G_{b_c}(x, x, x_0) \precsim G_{b_c}(x, x_0, y) = G_{b_c}(x_0, x, y) \prec r$$

is hold. By the property of the partial order $\prec, \precsim$ on $\mathbb{C}$, we have $G_{b_c}(x, x, x_0) \prec r$. Hence we get that $B_{G_b}(x_0, r) = \{x \in X |\ G_{b_c}(x_0, x, x) \prec r\}$.

Similarly,
$G_{b_c}(y, y, x_0) \precsim G_{b_c}(y, x_0, x) = G_{b_c}(x_0, x, y) \prec r$ and
$B_{G_b}(x_0, r) = \{y \in X |\ G_{b_c}(x_0, y, y) \prec r\}$.
So, we obtain that the desired result.

ii. Let $y \in B_{G_b}(x_0, r)$, hence $G_{b_c}(x_0, y, y) \prec r$. Let define that $G_{b_c}(x_0, y, y) = \frac{r-\delta}{s}$ and choose an element $x \in B_{G_b}(y, \delta)$, thus we have $G_{b_c}(y, x, x) \prec \delta$. We prove that $x \in B_{G_b}(x_0, r)$. By using $G_{b_c}5$ we have

$$\begin{aligned} G_{b_c}(x_0, x, x) &\precsim s\left[G_{b_c}(x_0, y, y) + G_{b_c}(y, x, x)\right] \\ &\prec s\left[\frac{r-\delta}{s} + \frac{\delta}{s}\right] = r. \end{aligned}$$

That is $x \in B_{G_b}(x_0, r)$. Hence, $B_{G_b}(y, \delta) \subseteq B_{G_b}(x_0, r)$. ∎

Proposition 4. Every complex valued G_b-metric space (X, G_{b_c}) is a topological space.

Proof. It follows from (ii) of Proposition 3 that the family of all open balls

$$\beta = \{B_{G_b}(x, r) : x \in X, 0 \prec r \in \mathbf{C}\}$$

is the base of a topology

$$\tau^r_{G_b} = \{U \subset X : \forall x \in U, \exists V \in \beta, x \in V \subset U\} \cup \{\emptyset\}$$

on X, the complex valued G_b-metric topology. Now we prove the properties of being topology. It is obvious that $\emptyset, X \in \tau^r_{G_b}$. Let U and V be elements of $\tau^r_{G_b}$ and $x \in U \cap V$. Then one can find $0 \prec z_1, z_2 \in \mathbb{C}$ such that $x \in B_{G_b}(x, z_1) \subset U$ and $x \in B_{G_b}(x, z_2) \subset V$. By using Proposition 1, there is $0 \prec z \in \mathbb{C}$ such that $z_1 \prec z$ and $z_2 \prec z$. Then

$$x \in B_{G_b}(x, z) \subset B_{G_b}(x, z_1) \cap B_{G_b}(x, z_2) \subset U \cap V.$$

Hence $U \cap V \in \tau^r_{G_b}$. Suppose that there are $U_i \in \tau^r_{G_b}$ and $x \in \bigcup_{i \in I} U_i$, for every $i \in I$, then there exists at least $i_0 \in I$ such that $x \in U_{i_0}$. So there exists $0 \prec z$ such that

$$x \in B_{G_b}(x, z) \subset U_{i_0} \subset \bigcup_{i \in I} U_i.$$

This implies that $\bigcup_{i \in I} U_i \in \tau^r_{G_b}$. Therefore, (X, G_{b_c}) is a topological space. ∎

Let X be a complex valued G_b-metric space. A sequence (x_n) in X is said to be Cauchy sequence, if for each $c \in \mathbb{C}$, with $0 \prec c$, there exists a positive integer n_0 such that, for all $m, n, l \geq n_0$, $G_{b_c}(x_n, x_m, x_l) \prec c$. The sequence (x_n) is convergent to a point $x \in X$ if for each $c \in \mathbb{C}$, there exists a positive integer n_0 such that, for all $m, n \geq n_0$, $G_{b_c}(x_n, x_m, x) \prec c$. A complex valued G_b-metric space X is called complete if every Cauchy sequence is convergent in X.

Now, we give some lemmas and propositions associated with convergence and Cauchy sequence of this space.

Lemma 2. Let (X, G_{b_c}) be a complex valued G_b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|G_{b_c}(x_n, x_m, x)| \to 0$ as $n \to +\infty$.

Proof. Suppose that $\{x_n\}$ is convergent to x. For a given real number $\epsilon > 0$, let

$$c = \frac{\varepsilon}{\sqrt{2}} + i\frac{\varepsilon}{\sqrt{2}}.$$

Then $0 \prec c \in \mathbb{C}$ and there is a natural number k such that $G_{b_c}(x_n, x_m, x) \prec c$ for all $m, n \geq k$. Therefore $|G_{b_c}(x_n, x_m, x)| < |c| = \epsilon$ for all $m, n \geq k$. It follows that $|G_{b_c}(x_n, x_m, x)| \to 0$ as $m, n \to \infty$.
Conversely, suppose that $|G_{b_c}(x_n, x_m, x)| \to 0$ as $m, n \to \infty$. Then given $c \in \mathbb{C}$ with $0 \prec c$, there exists a real number $\delta > 0$ such that for $z \in \mathbb{C}$, $|z| < \delta$ implies $z \prec c$. For this δ, there is a natural number k such that $|G_{b_c}(x_n, x_m, x)| < \delta$ for all $n, m \geq k$. This means that $G_{b_c}(x_n, x_m, x) \prec c$ for all $m, n \geq k$. Hence $\{x_n\}$ is convergent to x. ∎

Lemma 3. Let (X, G_{b_c}) be a complex valued G_b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|G_{b_c}(x_n, x_m, x_l)| \to 0$ as $n \to +\infty$.

Proof. Suppose that $\{x_n\}$ is a Cauchy sequence. For a given real number $\epsilon > 0$, let

$$c = \frac{\varepsilon}{\sqrt{2}} + i\frac{\varepsilon}{\sqrt{2}}.$$

Then $0 \prec c \in \mathbb{C}$ and there is a natural number k such that $G_{b_c}(x_n, x_m, x_l) \prec c$ for all $m, n, l \geq k$. Therefore $|G_{b_c}(x_n, x_m, x_l)| < |c| = \epsilon$ for all $m, n, l \geq k$. It follows that $|G_{b_c}(x_n, x_m, x_l)| \to 0$ as $m, n, l \to \infty$.
Conversely, suppose that $|G_{b_c}(x_n, x_m, x_l)| \to 0$ as $m, n, l \to \infty$. Then given $c \in \mathbb{C}$ with $0 \prec c$, there exists a real number $\delta > 0$ such that for $z \in \mathbb{C}$, $|z| < \delta$ implies $z \prec c$. For this δ, there is a natural number k such that $|G_{b_c}(x_n, x_m, x_l)| < \delta$ for all $n, m, l \geq k$. This means that $G_{b_c}(x_n, x_m, x_l) \prec c$ for all $m, n, l \geq k$. Hence $\{x_n\}$ is a Cauchy sequence. ∎

From proposition 1, lemma 2 and lemma 3, the following propositions are easily given.

Proposition 5. Let X be a complex valued G_b-metric space and $\{x_n\}$ be a sequence in X. Then the followings are equivalent:
i. the sequence $\{x_n\}$ is Cauchy;
ii. for any $0 \prec c$, there exists $n_0 \in \mathbb{N}$ such that $G_{b_c}(x_n, x_m, x_m) \prec c$, for all $m, n \geq n_0$.

Proof. **i** $\Rightarrow$ **ii**) Suppose that the sequence $\{x_n\}$ is Cauchy. Then for each $c \in \mathbb{C}$, with $0 \prec c$, there exists a positive integer n_0 such that, for all $m, n, l \geq n_0$, $G_{b_c}(x_n, x_m, x_l) \prec c$. From $G_{b_c}3$ with $x_m \neq x_l$, we have

$G_{b_c}(x_n, x_m, x_m) \precsim G_{b_c}(x_n, x_m, x_l) \prec c.$

So, we obtain that $G_{b_c}(x_n, x_m, x_m) \prec c$.
ii $\Rightarrow$ **i**) Let for any $0 \prec c$, there exists $n_0 \in \mathbb{N}$ such that $G_{b_c}(x_n, x_m, x_m) \prec \frac{c}{2s}$, and $G_{b_c}(x_l, x_m, x_m) \prec \frac{c}{2s}$, for all $m, n, l \geq n_0$. Using $G_{b_c}5$, we get that

$G_{b_c}(x_n, x_m, x_l) \precsim s\left[G_{b_c}(x_n, x_m, x_m) + G_{b_c}(x_m, x_m, x_l)\right] \prec \dfrac{c}{2s} + \dfrac{c}{2s} = c.$

This means that the sequence $\{x_n\}$ is Cauchy. ∎

Proposition 6. Let X be a complex valued G_b-metric space and $\{x_n\}$ be a sequence in X. Then the followings are equivalent:
i. $\{x_n\}$ is convergent to x;
ii. $|G_{b_c}(x_n, x_n, x)| \to 0$ as $n \to +\infty$;
iii. $|G_{b_c}(x_n, x, x)| \to 0$ as $n \to +\infty$.

Proof. **i** $\Rightarrow$ **ii**) Suppose that $\{x_n\}$ is convergent to x. So, by the Lemma 2, $|G_{b_c}(x_n, x_m, x)| \to 0, (n \to +\infty)$. From $G_{b_c}3$ with $x_n \neq x_m$, $G_{b_c}(x_n, x_n, x) \precsim G_{b_c}(x_n, x_m, x)$ which implies that $|G_{b_c}(x_n, x_n, x)| \leq |G_{b_c}(x_n, x_m, x)|$. Since $|G_{b_c}(x_n, x_m, x)| \to 0$, $(n \to +\infty)$, we obtain that $|G_{b_c}(x_n, x_n, x)| \to 0$.
ii $\Rightarrow$ **iii**) Let $|G_{b_c}(x_n, x_n, x)| \to 0$ as $n \to +\infty$. Using (iii) of proposition 1, we get that $G_{b_c}(x_n, x, x) \precsim 2sG_{b_c}(x, x_n, x_n)$ and $|G_{b_c}(x_n, x, x)| \leq 2s|G_{b_c}(x, x_n, x_n)|$. As $n \to +\infty$, the desired result is obtained.
iii $\Rightarrow$ **i**)As in the proof of previous conditions, this part is easily proved. ∎

Definition 9. Let (X, G_{b_c}) be complex valued G_b-metric space and let $f : X \to X$ be a mapping. Then f is said to be sequentially convergent if the sequence $\{x_n\}$ in X is convergent whenever $\{fx_n\}$ is convergent.

4. Common Fixed Point Theorems in Complex Valued G_b-Metric Space

In this section, we study the existence and uniqueness of fixed point in complex valued G_b-metric space.

Theorem 1. Let (X, G_{b_c}) be a complete complex valued G_b-metric space and $F : X \times X \to X$ and $g : X \to X$ be two mappings such that

$$G_{b_c}(F(x,y), F(u,v), F(w,z)) \precsim \alpha \frac{[G_{b_c}(gx, gu, gw) + G_{b_c}(gy, gv, gz)]}{2} \tag{4.1}$$
$$+\beta \frac{G_{b_c}(gx, gx, F(x,y))\, G_{b_c}(gu, gu, F(u,v))\, G_{b_c}(gw, gw, F(w,z))}{[G_{b_c}(gx, gu, F(w,z)) + G_{b_c}(gx, gw, F(u,v)) + G_{b_c}(gu, gw, F(x,y))]^2}$$

satisfies $x, y, u, v \in X$ where $0 < s^4(\alpha + \beta) < 1$. Assume that F and g satisfy the following conditions:
i. $F(X \times X) \subseteq g(X)$;
ii. $g(X)$ is complete;
iii. g is continuous and commutes with F.
Then F and g have a unique common coupled fixed point.

Proof. Choose x_0, y_0 as an arbitrary point of X. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$. Again since $F(X \times X) \subseteq g(X)$, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. Continuing this process, we can obtain two sequences (x_n) and (y_n) in X such that $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$. However, we will use the notation such that $F_n = F(x_n, y_n)$. For $n = 0, 1, \ldots$ using (1) we get

$$G_{b_c}(gx_{n-1}, gx_n, gx_n) = G_{b_c}(F(x_{n-2}, y_{n-2}), F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}))$$

$$\precsim \alpha \frac{[G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1}) + G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})]}{2}$$

$$+\beta \frac{G_{b_c}(gx_{n-2}, gx_{n-2}, F_{n-2}) G_{b_c}(gx_{n-1}, gx_{n-1}, F_{n-1}) G_{b_c}(gx_{n-1}, gx_{n-1}, F_{n-1})}{[G_{b_c}(gx_{n-2}, gx_{n-1}, F_{n-1}) + G_{b_c}(gx_{n-2}, gx_{n-1}, F_{n-1}) + G_{b_c}(gx_{n-1}, gx_{n-1}, F_{n-2})]^2}$$

$$= \alpha \frac{[G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1}) + G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})]}{2}$$

$$+\beta \frac{G_{b_c}(gx_{n-2}, gx_{n-2}, gx_{n-1}) G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n) G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n)}{[G_{b_c}(gx_{n-2}, gx_{n-1}, gx_n) + G_{b_c}(gx_{n-2}, gx_{n-1}, gx_n) + G_{b_c}(gx_{n-1}, gx_{n-1}, gx_{n-1})]^2}.$$

So, we have that

$$|G_{b_c}(gx_{n-1}, gx_n, gx_n)| \leq \alpha \frac{[|G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})| + |G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})|]}{2}$$

$$+\beta \frac{|G_{b_c}(gx_{n-2}, gx_{n-2}, gx_{n-1})| \, |G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n)| \, |G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n)|}{[2 \, |G_{b_c}(gx_{n-2}, gx_{n-1}, gx_n)|]^2}.$$

Since $|G_{b_c}(gx_n, gx_{n-1}, gx_{n-1})| \leq s \, |G_{b_c}(gx_n, gx_{n-1}, gx_{n-2})|$ and $|G_{b_c}(gx_n, gx_{n-2}, gx_{n-2})| \leq 2s \, |G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})|$, we obtain that

$$|G_{b_c}(gx_{n-1}, gx_n, gx_n)| \leq \alpha \frac{[|G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})| + |G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})|]}{2}$$

$$+\beta \frac{2s \, |G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})| \, |G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n)| \, |G_{b_c}(gx_{n-1}, gx_{n-1}, gx_n)|}{\left[\frac{2}{s} \, |G_{b_c}(gx_n, gx_{n-1}, gx_{n-1})|\right]^2}$$

which implies that

$$|G_{b_c}(gx_{n-1}, gx_n, gx_n)| \leq \alpha \frac{[|G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})| + |G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})|]}{2}$$

$$+\frac{\beta s^3}{2} \, |G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})|.$$

Similarly, one can show that

$$|G_{b_c}(gy_{n-1}, gy_n, gy_n)| \leq \alpha \frac{[|G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})| + |G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})|]}{2}$$

$$+\frac{\beta s^3}{2} \, |G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})|.$$

Then,

$$a_n := |G_{b_c}(gx_{n-1}, gx_n, gx_n)| + |G_{b_c}(gy_{n-1}, gy_n, gy_n)|$$

$$\leq \frac{2\alpha + \beta s^3}{2} \left[|G_{b_c}(gx_{n-2}, gx_{n-1}, gx_{n-1})| + |G_{b_c}(gy_{n-2}, gy_{n-1}, gy_{n-1})|\right]$$

$$= k a_{n-1}$$

where $k = \frac{2\alpha + \beta s^3}{2}$ for all $n \in \mathbf{N}$. Thus, we have

$$a_n \leq k a_{n-1} \leq k^2 a_{n-2} \leq \ldots \leq k^n a_0.$$

Now, we will demonstrate that (gx_n) and (gy_n) are Cauchy sequences. Let $m, n \in \mathbf{N}$ with $m > n$. By using the rectangle inequality, we have

$$
\begin{aligned}
&|G_{b_c}(gx_{n-1}, gx_m, gx_m)| + |G_{b_c}(gy_{n-1}, gy_m, gy_m)| \\
&\leq s\left[\left(|G_{b_c}(gx_{n-1}, gx_n, gx_n)| + |G_{b_c}(gx_n, gx_m, gx_m)|\right)\right. \\
&\quad \left.+ \left(|G_{b_c}(gy_{n-1}, gy_n, gy_n)| + |G_{b_c}(gy_n, gy_m, gy_m)|\right)\right] \\
&= s\left[\left(|G_{b_c}(gx_{n-1}, gx_n, gx_n)| + |G_{b_c}(gy_{n-1}, gy_n, gy_n)|\right)\right. \\
&\quad \left.+ \left(|G_{b_c}(gx_n, gx_m, gx_m)| + |G_{b_c}(gy_n, gy_m, gy_m)|\right)\right] \\
&\leq s\left(|G_{b_c}(gx_{n-1}, gx_n, gx_n)| + |G_{b_c}(gy_{n-1}, gy_n, gy_n)|\right) + s^2\left[\left(|G_{b_c}(gx_{n-1}, gx_n, gx_n)|\right.\right. \\
&+ \left.|G_{b_c}(gy_{n-1}, gy_n, gy_n)|\right) + \left.\left(|G_{b_c}(gx_{n+1}, gx_m, gx_m)| + |G_{b_c}(gy_{n+1}, gy_m, gy_m)|\right)\right] \\
&\leq \\
&\vdots \\
&\leq sa_n + s^2 a_{n+1} + s^3 a_{n+2} + \ldots + s^{m-n} a_{m-n-1} + s^{m-n+1} a_{m-n} \\
&\leq sk^n a_0 + s^2 k^{n+1} a_0 + s^3 k^{n+2} a_0 + \ldots + s^{m-n} k^{m-1} a_0 + s^{m-n+1} k^m a_0 \\
&\leq sk^n a_0 \left(1 + sk + s^2 k^2 + \ldots\right)
\end{aligned}
$$

since $sk < 1$, $\frac{sk^n a_0}{1-sk} \to 0$. Thus, $|G_{b_c}(gx_{n-1}, gx_m, gx_m)| \to 0$ and $|G_{b_c}(gy_{n-1}, gy_m, gy_m)| \to 0$. This means that (gx_n) and (gy_n) are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, (gx_n) and (gy_n) are convergent to $x \in X$ and $y \in X$, respectively.

As g is continuous, we get that (ggx_n) is convergent to gx and (ggy_n) is convergent to gy. Also, since F and g commute, we obtain

$$ggx_{n+1} = g(F(x_n, y_n)) = F(gx_n, gy_n)$$

and

$$ggy_{n+1} = g(F(y_n, x_n)) = F(gy_n, gx_n).$$

Thus

$$
\begin{aligned}
&G_{b_c}(ggx_{n+1}, F(x,y), F(x,y)) = G_{b_c}(F(gx_n, gy_n), F(x,y), F(x,y)) \\
&\precsim \alpha \frac{[G_{b_c}(ggx_n, gx, gx) + G_{b_c}(gy_n, gy, gy)]}{2} \\
&+\beta \frac{G_{b_c}(ggx_n, ggx_n, ggx_{n+1}) G_{b_c}(gx, gx, F(x,y)) G_{b_c}(gx, gx, F(x,y))}{[G_{b_c}(ggx_n, gx, F(x,y)) + G_{b_c}(ggx_n, gx, F(x,y)) + G_{b_c}(gx, gx, ggx_{n+1})]^2}
\end{aligned}
$$

so that

$$
\begin{aligned}
&|G_{b_c}(ggx_{n+1}, F(x,y), F(x,y))| \leq \alpha \frac{[|G_{b_c}(ggx_n, gx, gx)| + |G_{b_c}(gy_n, gy, gy)|]}{2} \\
&+\beta \frac{|G_{b_c}(ggx_n, ggx_n, ggx_{n+1})| |G_{b_c}(gx, gx, F(x,y))| |G_{b_c}(gx, gx, F(x,y))|}{[|G_{b_c}(ggx_n, gx, F(x,y))| + |G_{b_c}(ggx_n, gx, F(x,y))| + |G_{b_c}(gx, gx, ggx_{n+1})|]^2}.
\end{aligned}
$$

Letting $n \to \infty$, we get that

$$
\begin{aligned}
&|G_{b_c}(gx, F(x,y), F(x,y))| \leq \alpha \frac{[|G_{b_c}(gx, gx, gx)| + |G_{b_c}(gy, gy, gy)|]}{2} \\
&+\beta \frac{|G_{b_c}(gx, gx, gx)| |G_{b_c}(gx, gx, F(x,y))| |G_{b_c}(gx, gx, F(x,y))|}{[|G_{b_c}(gx, gx, F(x,y))| + |G_{b_c}(gx, gx, F(x,y))| + |G_{b_c}(gx, gx, gx)|]^2}.
\end{aligned}
$$

Consequently, from the above inequality we have $|G_{b_c}(gx, F(x,y), F(x,y))| = 0$ which implies that $gx = F(x,y)$. Similarly, we may show that $gy = F(y,x)$. Now, we will show that $x = gx$ and $y = gy$. Thus,

$$\begin{aligned} G_{b_c}(x, gx, gx) &\precsim s\left[G_{b_c}(x, gx_{n+1}, gx_{n+1}) + G_{b_c}(gx_{n+1}, gx, gx)\right] \\ &= s\left[G_{b_c}(x, gx_{n+1}, gx_{n+1}) + G_{b_c}(F(x_n, y_n), F(x,y), F(x,y))\right] \\ &\leq s\Bigg[G_{b_c}(x, gx_{n+1}, gx_{n+1}) + \alpha\frac{[G_{b_c}(gx_n, gx, gx) + G_{b_c}(gy_n, gy, gy)]}{2} \\ &\quad + \beta\frac{G_{b_c}(gx_n, gx_n, gx_{n+1})\,G_{b_c}(gx, gx, gx)\,G_{b_c}(gx, gx, gx)}{[G_{b_c}(gx_n, gx, gx) + G_{b_c}(gx_n, gx, gx) + G_{b_c}(gx, gx, gx_{n+1})]^2}\Bigg] \end{aligned}$$

which implies that

$$\begin{aligned} &\leq s\Bigg[|G_{b_c}(x, gx_{n+1}, gx_{n+1})| + \alpha\frac{[|G_{b_c}(gx_n, gx, gx)| + |G_{b_c}(gy_n, gy, gy)|]}{2} \\ &\quad + \beta\frac{|G_{b_c}(gx_n, gx_n, gx_{n+1})|\,|G_{b_c}(gx, gx, gx)|\,|G_{b_c}(gx, gx, gx)|}{[|G_{b_c}(gx_n, gx, gx)| + |G_{b_c}(gx_n, gx, gx)| + |G_{b_c}(gx, gx, gx_{n+1})|]^2}\Bigg]. \end{aligned}$$

Taking the limit as $n \to \infty$ and since $gx = F(x,y)$, we get that

$$|G_{b_c}(x, gx, gx)| \leq s\alpha\frac{[|G_{b_c}(x, gx, gx)| + |G_{b_c}(y, gy, gy)|]}{2}.$$

Similarly, we may show that

$$|G_{b_c}(y, gy, gy)| \leq s\alpha\frac{[|G_{b_c}(x, gx, gx)| + |G_{b_c}(y, gy, gy)|]}{2}.$$

Thus,

$$|G_{b_c}(x, gx, gx)| + |G_{b_c}(y, gy, gy)| \leq s\alpha\left[|G_{b_c}(x, gx, gx)| + |G_{b_c}(y, gy, gy)|\right].$$

Because $0 < s\alpha < 1$, the last inequality occurs only if $|G_{b_c}(x, gx, gx)| = 0$ and $|G_{b_c}(y, gy, gy)| = 0$. Hence $x = gx$ and $y = gy$. Thus we get

$$x = gx = F(x,y) \quad \text{and} \quad y = gy = F(y,x).$$

So, (x,y) is a common coupled fixed point of the mappings F and g. If there exists another common coupled fixed point $(u,v) \in X \times X$ with $(x,y) \neq (u,v)$ such that

$$u = gu = F(u,v) \quad \text{and} \quad v = gv = F(v,u).$$

Using inequality (1),

$$\begin{aligned} G_{b_c}(x, u, u) &= G_{b_c}(F(x,y), F(u,v), F(u,v)) \\ &\precsim \alpha\frac{[G_{b_c}(gx, gu, gu) + G_{b_c}(gy, gv, gv)]}{2} \\ &\quad + \beta\frac{G_{b_c}(gx, gx, F(x,y))\,G_{b_c}(gu, gu, F(u,v))\,G_{b_c}(gu, gu, F(u,v))}{[G_{b_c}(gx, gu, F(u,v)) + G_{b_c}(gx, gu, F(u,v)) + G_{b_c}(gx, gu, F(u,v))]^2} \\ &\precsim \alpha\frac{[G_{b_c}(x, u, u) + G_{b_c}(y, v, v)]}{2} \\ &\quad + \beta\frac{G_{b_c}(x, x, x)\,G_{b_c}(u, u, u)\,G_{b_c}(u, u, u)}{[G_{b_c}(x, u, u) + G_{b_c}(x, u, u) + G_{b_c}(x, u, u)]^2} \end{aligned}$$

so that

$$|G_{b_c}(x,u,u)| \le \alpha \frac{[|G_{b_c}(x,u,u)| + |G_{b_c}(y,v,v)|]}{2}.$$

Similarly, we can easily prove that

$$|G_{b_c}(y,v,v)| \le \alpha \frac{[|G_{b_c}(x,u,u)| + |G_{b_c}(y,v,v)|]}{2}.$$

If we add above inequalities, we get

$$[|G_{b_c}(x,u,u)| + |G_{b_c}(y,v,v)|] \le \alpha [|G_{b_c}(x,u,u)| + |G_{b_c}(y,v,v)|]$$

which is a contradiction because $0 < \alpha < 1$. Thus, we get $x = u$ and $y = v$, which proves the uniqueness of common coupled fixed point of F and g.

Corollary 1. Let (X, G_{b_c}) be a complete complex valued G_b-metric space and $F : X \times X \to X$ be a mapping such that

$$G_{b_c}(F(x,y), F(u,v), F(u,v)) \precsim \alpha \frac{[G_{b_c}(x,u,w) + G_{b_c}(y,v,z)]}{2}$$

$$+\beta \frac{G_{b_c}(x,x,F(x,y))\, G_{b_c}(u,u,F(u,v))\, G_{b_c}(w,w,F(w,z))}{[G_{b_c}(x,u,F(w,z)) + G_{b_c}(x,w,F(u,v)) + G_{b_c}(u,w,F(x,y))]^2} \tag{4.2}$$

satisfies $x, y, u, v \in X$ where $0 < s(\alpha + \beta) < 1$. Then F has a unique coupled fixed point.

Proof. By setting $g = I$, the proof is deduced.

Now, we present an example to illustrate Theorem 1.

Example 3. Let $X = [0,1]$, $G_{b_c} : X \times X \times X \to \mathbb{C}$ be defined by

$$G_{b_c}(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\} + i \max\{d(x,y), d(y,z), d(z,x)\},$$

with $d(x,y) = |x-y|^2$ where $s = 2$. Define $F : X \times X \to X$ and $g : X \to X$ as

$$F(x,y) = \frac{x-y}{12}, \qquad g(x) = \frac{x}{3}$$

for all $x, y \in X$. Suppose that $\alpha = \frac{1}{4}$ and $\beta \in [0,1)$ with $0 < s(\alpha+\beta) < 1$. Now, we have

$$G_{b_c}(F(x,y), F(u,v), F(w,z)) = G_{b_c}\left(\frac{x-y}{6}, \frac{u-v}{6}, \frac{w-z}{6}\right)$$

$$= \max\left\{\left|\frac{x-y-(u-v)}{6}\right|^2, \left|\frac{u-v-(w-z)}{6}\right|^2, \left|\frac{x-y-(w-z)}{6}\right|^2\right\}$$

$$+i \max\left\{\left|\frac{x-y-(u-v)}{6}\right|^2, \left|\frac{u-v-(w-z)}{6}\right|^2, \left|\frac{x-y-(w-z)}{6}\right|^2\right\}.$$

Let $\max = \left|\frac{x-y-(u-v)}{6}\right|^2$, then we get

$$G_{b_c}\left(\frac{x-y}{6}, \frac{u-v}{6}, \frac{w-z}{6}\right) = \left|\frac{x-y-(u-v)}{6}\right|^2 + i\left|\frac{x-y-(u-v)}{6}\right|^2$$

$$\leq \frac{(|x-y|+|u-v|)^2}{144}+i\frac{(|x-y|+|u-v|)^2}{144}$$
$$\leq \frac{2}{16}\left[\frac{|x-y|^2+i|x-y|^2}{9}+\frac{|u-v|^2+i|u-v|^2}{9}\right]$$
$$=\frac{1}{4}\frac{[G_{b_c}(gx,gu,gw)+G_{b_c}(gy,gv,gz)]}{2}$$
$$\precsim \frac{1}{4}\frac{[G_{b_c}(gx,gu,gw)+G_{b_c}(gy,gv,gz)]}{2}$$
$$+\beta\frac{G_{b_c}(gx,gx,F(x,y))\,G_{b_c}(gu,gu,F(u,v))\,G_{b_c}(gw,gw,F(w,z))}{[G_{b_c}(gx,gu,F(w,z))+G_{b_c}(gx,gw,F(u,v))+G_{b_c}(gu,gw,F(x,y))]^2}.$$

Obviously, all the conditions of Theorem 1 are satisfied. Notice that $(0,0)$ is a common coupled fixed point of F and g.

Conflict of Interests

The author declare that there is no conflict of interests regarding the publication of this paper.

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