



A COMMON FIXED POINT THEOREM IN FUZZY METRIC SPACES UNDER THE PROPERTY (CLR_g)

Praveen Kumar Sharma*

*Department of Mathematics, Institute of Applied Sciences and Humanities,
GLA UNIVERSITY, MATHURA-281406, INDIA
E-mail: praveen.jan1980@rediffmail.com*

**Corresponding author.*

Abstract In this note we generalize the results of M. Abbas, I. Altun, D. Gopal [M. Abbas, I. Altun, D. Gopal, Common fixed point theorems for non compatible mappings in fuzzy metric spaces, Bulletin of Mathematical analysis and Applications, 1(2) (2009), 47-56] using weakly compatible mappings under the property (CLR_g). We also give an example in support main result.

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1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [1] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. In the last two decades there has been a tremendous development and growth in fuzzy mathematics. Kramosil and Michalek [2] introduced the notion of a fuzzy metric space by generalizing the concept of the probabilistic metric space to the fuzzy situation. George and Veeramani [3] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [2]. There are many view points of the notion of the metric space in fuzzy topology for example we can refer to Kaleva and Seikkala [4], Kramosil and Michalek [2] and George and Veeramani [3]. Mishra et al. [5] introduced the concept of compatible mappings in fuzzy metric spaces which was further generalised by Singh and Jain [6]. In 2002, Aamri and El- Moutawakil [7] defined the notion of property (E-A) in metric spaces for self mappings which contained the class of non compatible mappings in metric spaces. Pant and Pant [8] proved some sommon fixed points for a pair of maps under the notion of property (E.A.) and non-compatible maps. Sharma and Bamboria [9] defined a property (S-B) in fuzzy metric spaces for self

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maps and obtained some common fixed point theorems in IFMS for such mappings under strict contractive conditions, further Sharma and Sharma [10,11] proved some common fixed point theorems in IFMS by using this concept. The class of (S-B) maps contains the class of non compatible maps.

Most recently, Sintunavarat and Kumam [12] defined the notion of "common limit in the range" property or CLR property in fuzzy metric spaces. It is observed that the notion of CLR property never requires the condition of the closedness of the subspace while (E-A), common (E-A) and (S-B) property require this condition for the existence of the fixed point and hence, now a days, authors are giving much attention to this property for generalizing the results present in the literature. Works noted in the references [13-19] are some examples. Popa [20, 21] introduced the idea of implicit function to prove a common fixed point theorem in metric spaces. Jain [22] further extended the result of Popa [20, 21] in fuzzy metric spaces. Afterwards, implicit relations are used as a tool for finding common fixed point of contraction maps (see, [23-30]).

In [31] abbas et al obtained common fixed point of mappings satisfying generalized contractive type conditions without exploiting the notion of continuity in the setting of fuzzy metric spaces, they proved two common fixed point theorems for (EA) and common (EA) property respectively.

In our main result we generalize the results of Abbas et al. [31] by using the property (CLRg) and relaxing (EA), common (EA) properties and many conditions involved. We prove our main result without any requirement of completeness of the whole space (or closedness of any subspace).

2. PRELIMINARIES

For proving our main result we need the following definitions:

Definition 2.1: ([1]) A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2: ([32]) A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -norm if $([0, 1], *)$ is a topological abelian monoid with unit 1 s.t. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3: ([3]) The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t -norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions:

$$(FM-1) \quad M(x, y, 0) > 0,$$

$$(FM-2) \quad M(x, y, t) = 1 \text{ if } x = y,$$

$$(FM-3) \quad M(x, y, t) = M(y, x, t),$$

$$(FM-4) \quad M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),$$

$$(FM-5) \quad M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1] \text{ is continuous, for all } x, y, z \in X \text{ and } s, t > 0.$$

Throughout this paper, we consider M to be a fuzzy metric space with condition:

$$(FM-6) \quad \lim_{t \rightarrow \infty} M(x, y, t) = 1 \text{ for all } x, y \in X \text{ and } t > 0.$$

Definition 2.4: ([3]) Let $(X, M, *)$ be fuzzy metric space. A sequence $\{x_n\}$ in X is said to be

(i) Convergent to a point $x \in X$, if $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ for all $t > 0$;

(ii) Cauchy sequence if $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$, for all $t > 0$ and $p > 0$.

Definition 2.5: ([3]) A fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.6: ([33]) $M(x, y, \cdot)$ is non-decreasing for all $x, y \in X$.

Lemma 2.7: ([33]) Let $x_n \rightarrow x$ and $y_n \rightarrow y$, then

(i) $\lim_{n \rightarrow \infty} M(x_n, y_n, t) \geq M(x, y, t)$, for all $t > 0$,

(ii) $\lim_{n \rightarrow \infty} M(x_n, y_n, t) = M(x, y, t)$, for all $t > 0$, if $M(x, y, t)$ is continuous.

Lemma 2.8: ([5]) If for all $x, y \in X, t > 0$ and for a number $k \in (0, 1)$; $M(x, y, kt) \geq M(x, y, t)$, then $x = y$.

Definition 2.9: ([5]) Let A and B be maps from a FM-space (X, M, \cdot) into itself.

The maps A and B are said to be compatible (or asymptotically commuting), if for all $t, \lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z$ for some $z \in X$.

From the above definition it is inferred that A and B are non-compatible maps from a FM-space (X, M, \cdot) into itself if $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n$ for some $z \in X$, but either $\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) \neq 1$ or the limit does not exist.

Definition 2.10: ([6]) Let A and B be maps from a FM-space (X, M, \cdot) into itself.

The maps are said to be weakly compatible if they commute at their coincidence points. Note that compatible mappings are weakly compatible but converse is not true in general.

Definition 2.11: ([8]) Let A and B be two self-maps of a FM-space (X, M, \cdot) .

We say that A and B satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X.$$

Note that weakly compatible and property (E.A.) are independent to each other (see [20], Example 2.2).

Definition 2.12. Mappings A, B, S and T on a fuzzy metric space $(X, M, *)$ are said to satisfy common (E.A.) property if there exists sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z \text{ for some } z \in X.$$

For more on (E.A) and common (E.A) properties, we refer to [7] and [34].

Definition 2.13: ([12]) Let (X, d) be a metric space. Two mappings $f : X \rightarrow X$ and $g : X \rightarrow X$ are said to satisfy property (CLRg) if there exists sequences $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} g(x_n) = g(p), \text{ for some } p \text{ in } X.$$

Similarly, we can have the property (CLRT) and the property (CLRS) if in the definition 2.13, the mapping $g : X \rightarrow X$ has been replaced by the mapping $T : X \rightarrow X$ and $S : X \rightarrow X$ respectively.

Definition 2.14. Let ψ a class of implicit relations be the set of all continuous functions $\phi : [0, 1] \times [0, 1] \rightarrow [0, 1]$ which are increasing in each coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$. For examples of implicit relations we refer to [35] and references there in.

3. MAIN RESULTS

In [31], M. Abbas, I. Altun, D. Gopal proved two theorems (theorem A theorem B) for property (E.A) and for common property (E.A) respectively, they proved the following:

Theorem A. Let $(X, M, *)$ be a fuzzy metric space. Let A, B, S and T be maps from X into itself with $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ and there exists a constant $k \in (0, \frac{1}{2})$ such that

$$M(Ax, By, kt) \geq \phi \left(M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2 - \alpha)t) \right), \quad (1)$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$ and $\phi \in \psi$.

Then A, B, S and T have a unique common fixed point in X provided the pair $\{A, S\}$ or $\{B, T\}$ satisfies (E.A) property, one of $A(X), T(X), B(X), S(X)$ is a closed subset of X and the pairs $\{B, T\}$ and $\{A, S\}$ are weakly compatible.

Theorem B. Let $(X, M, *)$ be a fuzzy metric space. Let A, B, S and T be maps from X into itself such that

$$M(Ax, By, kt) \geq \phi \left(M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2 - \alpha)t) \right), \quad (1)$$

for all $x, y \in X, k \in (0, \frac{1}{2}), \alpha \in (0, 2), t > 0$ and $\phi \in \psi$.

Then A, B, S and T have a unique common fixed point in X provided the pair $\{A, S\}$ and $\{B, T\}$ satisfy common (E.A) property, $T(X)$, and $S(X)$ are closed subset of X and the pairs $\{B, T\}$ and $\{A, S\}$ are weakly compatible.

We now generalize the theorems A and B as follows;

Theorem 3. Let $(X, M, *)$ be a fuzzy metric space with $a*b = \min\{a, b\}$. Let A, B, S, T be maps from X into itself satisfying the following conditions:

(3.1) $B(X) \subseteq S(X)$ and the pair (B, T) satisfies property (CLR_T) or $A(X) \subseteq T(X)$ and the pair (A, S) satisfies property (CLR_S)

(3.2) the pairs (A, S) and (B, T) are weakly compatible.

(3.3) there exists a constant $k \in (0, \frac{1}{2})$ such that

$$M(Ax, By, kt) \geq \phi \left(M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2 - \alpha)t) \right)$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$ and $\phi \in \psi$.

Then A, B, S, T have a unique common fixed point in X .

Proof. Assume that $B(X) \subseteq S(X)$ and the pair (B, T) satisfies property (CLR_T) , then there exists a sequence $\{x_n\}$ in X such that Bx_n and Tx_n converges to Tx , for some x in X as $n \rightarrow \infty$. Since $B(X) \subseteq S(X)$, so there exists a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$, hence $Sy_n \rightarrow Tx$ as $n \rightarrow \infty$.

We shall show that $\lim_{n \rightarrow \infty} Ay_n = Tx$. Let $\lim_{n \rightarrow \infty} Ay_n = z$. Taking $x = y_n, y = x_n$ and $\alpha = 1$ in (3.3), we have

$$M(Ay_n, Bx_n, kt) \geq \phi \left(M(Sy_n, Tx_n, t), M(Ay_n, Sy_n, t), M(Bx_n, Tx_n, t), \right. \\ \left. M(Ay_n, Tx_n, t), M(Bx_n, Sy_n, t) \right)$$

Taking $n \rightarrow \infty$, we have

$$M(z, Tx, kt) \geq \phi \left(M(Tx, Tx, t), M(z, Tx, t), M(Tx, Tx, t), M(z, Tx, t), \right. \\ \left. M(Bx_n, Bx_n, t) \right) \\ M(z, Tx, kt) \geq \phi \left(1, M(z, Tx, t), 1, M(z, Tx, t), 1 \right)$$

Since ϕ is increasing in each of its coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, so

$$M(z, Tx, kt) > M(z, Tx, t)$$

By lemma (2.8), which implies that $z = Tx$ or $\lim_{n \rightarrow \infty} Ay_n = Tx$.

Subsequently, we have Bx_n, Tx_n, Sy_n, Ay_n converges to z .

Now, we shall show that $Bx = z$.

Taking $x = y_n, y = x$ and $\alpha = 1$ in (3.3), we have

$$M(Ay_n, Bx, kt) \geq \phi \left(M(Sy_n, Tx, t), M(Ay_n, Sy_n, t), M(Bx, Tx, t), \right. \\ \left. M(Ay_n, Tx, t), M(Bx, Sy_n, t) \right)$$

Taking limit $n \rightarrow \infty$, we obtain

$$M(z, Bx, kt) \geq \phi \left(M(z, z, t), M(z, z, t), M(Bx, z, t), M(z, z, t), M(Bx, z, t) \right) \\ M(z, Bx, kt) \geq \phi \left(1, 1, M(z, Bx, t), 1, M(z, Bx, t) \right)$$

Since ϕ is increasing in each of its coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, so

$$M(z, Bx, kt) > M(z, Bx, t)$$

which implies that, $z = Bx$.

Hence, $z = Bx = Tx$.

Since, the pair (B, T) is weak compatible, it follows that $Bz = Tz$.

Also, since $B(X) \subseteq S(X)$, there exists some y in X such that $Bx = Sy (= z)$.

We next show that $Sy = Ay (= z)$.

Taking $y = x_n, x = y$ and $\alpha = 1$ in (3.3), we have

$$M(Ay, Bx_n, kt) \geq \phi \left(M(Sy, Tx_n, t), M(Ay, Sy, t), M(Bx_n, Tx_n, t), \right. \\ \left. M(Ay, Tx_n, t), M(Bx_n, Sy, t) \right)$$

Taking limit $n \rightarrow \infty$, we have

$$M(Ay, z, kt) \geq \phi\left(M(z, z, t), M(Ay, z, t), M(z, z, t), M(Ay, z, t), M(z, z, t)\right).$$

$$M(Ay, z, kt) \geq \phi\left(1, M(Ay, z, t), 1, M(Ay, z, t), 1\right).$$

Since ϕ is increasing in each of its coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, so

$$M(Ay, z, kt) > M(Ay, z, t)$$

which implies that $Ay = z$

which implies that $Ay = z = Sy$.

But the pair (A, S) is weakly compatible, it follows that $Az = Sz$.

Next, we claim that $Az = Bz$.

Taking $x = z, y = z$ and $\alpha = 1$ in (3.3), we have

$$M(Az, Bz, kt) \geq \phi\left(M(Sz, Tz, t), M(Az, Sz, t), M(Bz, Tz, t), M(Az, Tz, t), M(Bz, Sz, t)\right)$$

$$M(Az, Bz, kt) \geq \phi\left(M(Az, Bz, t), M(Az, Az, t), M(Bz, Bz, t), M(Az, Bz, t), M(Bz, Az, t)\right)$$

$$M(Az, Bz, kt) \geq \phi\left(M(Az, Bz, t), 1, 1, M(Az, Bz, t), M(Az, Bz, t)\right),$$

Since ϕ is increasing in each of its coordinate and $\phi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, so

$$M(Az, Bz, kt) > M(Az, Bz, t)$$

which implies that $Az = Bz$.

Hence, $Az = Bz = Sz = Tz$.

We now show that $z = Az$.

Taking $x = z, y = x$ and $\alpha = 1$ in (3.3), we have

$$M(Az, Bx, kt) \geq \phi\left(M(Sz, Tx, t), M(Az, Sz, t), M(Bx, Tx, t), M(Az, Tx, t), M(Bx, Sz, t)\right)$$

that is,

$$M(Az, z, kt) \geq \phi\left(M(Az, z, t), M(Az, Az, t), M(z, z, t), M(Az, z, t), M(z, Az, t)\right)$$

$$M(Az, z, kt) \geq \phi\left(M(Az, z, t), 1, 1, M(Az, z, t), M(Az, z, t)\right)$$

Since ϕ is increasing in each of its coordinate, so $M(Az, z, kt) > M(Az, z, t)$

which implies that $z = Az$.

Therefore, $z = Az = Bz = Sz = Tz$, that is z is the common fixed point of the maps A, B, S, T .

Uniqueness. The uniqueness of common fixed point of the mappings A, B, S and T be easily verified by using (3.3);

In fact, if w be another fixed point for mappings A, B, S and T then for $x = z, y = w$ and for $\alpha = 1$ in (3.3) we have,

$$M(z, w, kt) \geq \phi\left(M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)\right)$$

$$M(z, w, kt) \geq \phi\left(M(z, w, t), 1, 1, M(z, w, t), M(z, w, t)\right)$$

Since ϕ is increasing in each of its coordinate, so $M(z, w, kt) > M(z, w, t)$

Which implies that $z = w$.

Hence common fixed point is unique. This completely establishes the theorem.

Example 3.1. Let $X = [0, 1]$ and M be the usual fuzzy metric space on $(X, M, *)$ with minimum t -norm, defined by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases} \quad \text{for all } x, y \in X.$$

Then $(X, M, *)$ is a fuzzy metric space, where $*$ is continuous t -norm defined by $a * b = \min\{a, b\}$ for all a, b in $[0, 1]$.

Define the mappings $A, B, S, T : X \rightarrow X$ by $Ax = x/75, Bx = x/15, Sx = x/5, Tx = x$ respectively. Then, for some $k \in [1/15, 1)$, we have

$$M(Ax, By, kt) = \left[\frac{kt}{kt + |\frac{x}{75} - \frac{y}{15}|} \right] \geq \left[\frac{t}{t + |\frac{x}{5} - y|} \right]$$

$$= M(Sx, Ty, t) \geq \phi\left\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\right\} \text{ for } \alpha = 1.$$

Thus the condition (3.3) of theorem [3] is satisfied.

Further the pairs (A, S) and (B, T) are weakly compatible.

Also $B(X) = [0, 1/15] \subseteq [0, 1/5] = S(X)$.

Consider the sequence $\{x_n\} = \{\frac{1}{n}\}$ so that $\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Tx_n = 0 = T(0)$, hence the pair (B, T) satisfies property (CLR_T) .

Therefore all the conditions of theorem [3] are satisfied.

4. CONCLUSION

Theorem [3] is proved for weakly compatible mappings using the $(CLR_T)/(CLR_s)$ property in FM-space without any requirement of closedness of any subspace. An example [3.1] validates the theorem [3]. Theorem [3] improves and generalizes the results of Abbas et al. [31] and some earlier results.

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King Mongkuts University of Technology Thonburi (KMUTT)

126 Pracha Uthit Road, Bang Mod, Thung Khru, Bangkok, Thailand 10140

Website: <http://tacs.kmutt.ac.th/>Email: tacs@kmutt.ac.th