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COMMON FIXED POINTS FOR FOUR MAPS IN ORDERED FUZZY METRIC SPACES USING (ψ, ϕ, φ)-CONTRACTIONS WITH ADMISSIBLE FUNCTIONS

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Abstract In this paper, we prove a unique common fixed point theorem for two pairs of mappings satisfying (ψ, ϕ, φ) admissible contractive condition in partially ordered fuzzy metric spaces. Our result generalizes and improves results of Gregori and Sapena [6]and Gopal and Vetro [4]. We also give an example to support our theorem.

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1. Introduction

The concept of fuzzy sets was introduced initially by Zadeh[17] in 1965. George and Veeramani [3] modified the concept of fuzzy topological spaces induced by fuzzy metric introduced by Kramosil and Michalek [8] and Grabiec[5] and proved the contraction principle in the setting of fuzzy metric spaces. Many authors,for example,[2, 5, 6, 9, 10, 12, 15, 16] have proved fixed and common fixed point theorems in fuzzy metric spaces.
We donote $\mathcal{R}, \mathcal{R}^+$ and $\mathcal{N}$ for the sets of real numbers, non-negative real numbers and natural numbers respectively.Now, we give the following preliminaries.

Definition 1.1([14]). A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

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(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0, 1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Two typical examples of a continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$. We use the following definition due to George and Veeramani [3].

Definition 1.2([3]). A 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(5) $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the *open ball* $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by
$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$.

If $(X, M, *)$ is a fuzzy metric space, let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable.
A sequence $\{x_n\}$ in X converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$, for each $t > 0$. It is called a G-Cauchy sequence in the sense of [3] if $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$, for all $t > 0$ and each $p \in \mathcal{N}$. The fuzzy metric space $(X, M, *)$ is said to be *G-complete* if every G-Cauchy sequence is convergent.

Example 1.3. Let $X = \mathcal{R}$. Put $a * b = ab$ *or* $\min\{a, b\}$ for all $a, b \in [0, 1]$. For all $x, y \in X$,define $M(x, y, t) = \frac{t}{t+|x-y|}$ for $t > 0$ and $M(x, y, 0) = 0$.Then $(X, M, *)$ is a fuzzy metric space.

Example 1.4. Let $X = \mathcal{R}$. Put $a * b = ab$ for all $a, b \in [0, 1]$. For all $x, y \in X$,define $M(x, y, t) = e^{-\frac{|x-y|}{t}}$ for $t > 0$ and $M(x, y, 0) = 0$. Then $(X, M, *)$ is a fuzzy metric space.

Lemma 1.5.[5] Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.

Definition 1.6. Let $(X, M, *)$ be a fuzzy metric space. Then M is said to be *continuous* on $X^2 \times (0, \infty)$ if $\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$, whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$. i.e. $\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$
and $\lim_{n\to\infty} M(x, y, t_n) = M(x, y, t)$.

Lemma 1.7.([11]) Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.

Definition 1.8.([10]) Let $(x, M, *)$ be a fuzzy metric space and $f, S : X \to X$. The pair (f, S) is said to be compatible if $\lim_{n\to\infty} M(fSx_n, Sfx_n, t) = 1$ for every $t > 0$, whenever there exists a sequence $\{x_n\}$ in X such that $fx_n \to z$ and $Sx_n \to z$ as $n \to \infty$ for some $z \in X$.

Definition 1.9.([7])Let X be a non-empty set and $f, S : X \to X$.The pair (f, S) is said to be weakly compatible if $fSu = Sfu$ whenever $fu = Su$ for $u \in X$.
Samet et.al ([13]) introduced the notion of α- admissible mappings as follows

Definition 1.10. ([13]) Let X be a non empty set, $T : X \to X$ and
$\alpha : X \times X \to \mathcal{R}^+$ be mappings. Then T is called α- admissible if for all $x, y \in X$, we have

$\alpha(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$.
Some interesting examples of such mappings are given in ([13]).
Gopal and Vetro [4]defined the following

Definition 1.11. Let $(X, M, *)$ be a fuzzy metric space. The map $T : X \to X$ is α-admissible if there exists a function $\alpha : X \times X \times (0, \infty) \to \mathcal{R}^+$ such that $\alpha(x, y, t) \geq 1$ implies $\alpha(Tx, Ty, t) \geq 1$ for all $x, y \in X$ and for all $t > 0$.

Theorem 1.12.(*Theorem* 3.6, [4]) Let $(X, M, *)$ be a G-complete fuzzy metric space. Let $T : X \to X$ and $\alpha : X \times X \times (0, \infty) \to \mathcal{R}^+$ be satisfying
$(i)\alpha(x, y, t) \left(\frac{1}{M(Tx,Ty,t)} - 1 \right) \leq \phi \left(\frac{1}{M(x,y,t)} - 1 \right), \forall x, y \in X$ and $\forall t > 0$,
where $\phi : \mathcal{R}^+ \to \mathcal{R}^+$ is right continuous and $\phi(r) < r, \forall r > 0$,
(ii) T is α-admissible,
(iii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0, t) \geq 1, \forall t > 0$,
(iv) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}, t) \geq 1, \forall n \in \mathcal{N}$ and $\forall t > 0$ and $x_n \to x$, then $\alpha(x_n, x, t) \geq 1, \forall n \in \mathcal{N}$ and $\forall t > 0$.
Then T has a fixed point in X.

In this paper, we introduce α-admissible condition for two pairs of maps in fuzzy metric spaces as follows

Definition 1.13. Let $(X, M, *)$ be a fuzzy metric space and $f, g, S, T : X \to X$ be mappings and $\alpha : X \times X \times (0, \infty) \to \mathcal{R}^+$ be a function. We say that the pair(f, g) satisfies α-admissible condition with respect to the pair (S, T) if $\alpha(Sx, Ty, t) \geq 1$ implies $\alpha(fx, gy, t) \geq 1$ and $\alpha(Tx, Sy, t) \geq 1$ implies $\alpha(gx, fy, t) \geq 1$ $\forall x, y \in X$ and $\forall t > 0$.

Recently Abbas et al. [1] introduced the new concepts in a partially ordered set as follows

Definition 1.14.([1]) Let $(X, \preceq)$ be a partially ordered set and $f, g : X \to X$.
(i)f is said to be a dominating map if $x \preceq fx$.
(ii) f is said to be a weak annihilator of g if $fgx \preceq x$.

Using these concepts, we now prove a unique common fixed point theorem for four maps with α -admissible condtion in partially ordered fuzzy metric spaces.

2. Main Results

Theorem 2.1: Let $(X, M, *, \preceq)$ be a partially ordered G- complete fuzzy metric space and $f, g, S, T : X \to X$ and $\alpha : X \times X \times (0, \infty) \to \mathcal{R}^+$ be a function satisfying

(2.1.1) f and g are dominating maps and f and g are weak annihilators of T and S respectively,

(2.1.2) $f(X) \subseteq T(X)$, $g(X) \subseteq S(X)$,

(2.1.3) $\alpha(Sx, Ty, t)\ \psi \left(\frac{1}{M(fx,gy,t)} - 1 \right) \leq \phi \left(\frac{1}{m(x,y,t)} - 1 \right) - \varphi \left(\frac{1}{m(x,y,t)} - 1 \right)$

for all comparable elements $x, y \in X$, $\forall\ t > 0$, where
$m(x, y, t) = \min\{M(Sx, Ty, t), M(fx, Sx, t), M(gy, Ty, t)\}$ and
$\psi, \phi, \varphi : \mathcal{R}^+ \to \mathcal{R}^+$ are such that ψ is monotonically increasing and continuous and ϕ and φ are upper and lower semi continuous respectively with satisfying the folowing condition

$(A) : \psi(t) - \phi(t) + \varphi(t) > 0$ for all $t > 0$

(2.1.4) (f, g) is α-admissible w.r.to (S, T),

(2.1.5) $\alpha(Sx_1, fx_1, t) \geq 1$ and $\alpha(fx_1, Sx_1, t) \geq 1$ for some $x_1 \in X$ and $\forall t > 0$, (2.1.6)(a) S is continuous,the pair (f, S) is compatible and the pair (g, T) is weakly compatible and we assume $\alpha(Sy_{2n}, y_{2n-1}, t) \geq 1$, $\alpha(z, y_{2n-1}, t) \geq 1$, $\alpha(y_{2n}, Tz, t) \geq 1$ and $\alpha(z, z, t) \geq 1$ $\forall n \in \mathcal{N}$ and $\forall t > 0$ whenever there exists $\{y_n\}$ in X such that $\alpha(y_n, y_{n+1}, t) \geq 1$ and $\alpha(y_{n+1}, y_n, t) \geq 1$ $\forall n \in \mathcal{N}$ and for all $t > 0$ and $y_n \to z$ for some $z \in X$.

(or)

(2.1.6)(b) T is continuous,the pair (g, T) is compatible and the pair (f, S) is weakly compatible and we assume $\alpha(y_{2n}, Ty_{2n-1}, t) \geq 1$, $\alpha(y_{2n}, z, t) \geq 1$, $\alpha(Sz, y_{2n-1}, t) \geq 1$ and $\alpha(z, z, t) \geq 1$ $\forall n \in \mathcal{N}$ and $\forall t > 0$ whenever there exists $\{y_n\}$ in X such that $\alpha(y_n, y_{n+1}, t) \geq 1$ and $\alpha(y_{n+1}, y_n, t) \geq 1$ $\forall n \in \mathcal{N}$ and for all $t > 0$ and $y_n \to z$ for some $z \in X$.

(2.1.7) if for a non-decreasing sequence $\{x_n\}$ in X with $x_n \preceq y_n$,$\forall n \in \mathcal{N}$ and $y_n \to z$ implies $x_n \preceq z$,$\forall n \in \mathcal{N}$.

Then f, g, S and T have a common fixed point in X .

(2.1.8) Further if we assume that $\alpha(u, v, t) \geq 1$ $\forall$ $t > 0$ whenever u and v are common fixed points of f, g, S and T and the set of common fixed points of f, g, S and T is well ordered then f, g, S and T have unique common fixed point in X.

Proof. From (2.1.5), there exists $x_1 \in X$ such that $\alpha(Sx_1, fx_1, t) \geq 1$ and $\alpha(fx_1, Sx_1, t) \geq 1$, $\forall t > 0$.
From (2.1.2), we define the sequences $\{x_n\}$ and $\{y_n\}$ as
$y_{2n+1} = fx_{2n+1} = Tx_{2n+2}$, $y_{2n+2} = gx_{2n+2} = Sx_{2n+3}$, $n = 0, 1, 2, \cdots$.
Now

$$\begin{aligned}
\alpha(Sx_1, fx_1, t) \geq 1 \quad &\Rightarrow \alpha(Sx_1, Tx_2, t) \geq 1 \text{ from definition of } \{y_n\} \\
&\Rightarrow \alpha(fx_1, gx_2, t) \geq 1, \text{ from (2.1.4), } i.e\ \alpha(y_1, y_2, t) \geq 1 \\
&\Rightarrow \alpha(Tx_2, Sx_3, t) \geq 1 \text{ from definition of } \{y_n\} \\
&\Rightarrow \alpha(gx_2, fx_3, t) \geq 1, \text{ from (2.1.4), } i.e\ \alpha(y_2, y_3, t) \geq 1.
\end{aligned}$$

Continuing in this way, we have

$$\alpha(y_n, y_{n+1}, t) \geq 1, \ \forall\ n \in \mathcal{N} \ and \ \forall\ t > 0 \tag{2.1}$$

Similarly by using $\alpha(fx_1, Sx_1, t) \geq 1$, we can show that

$$\alpha(y_{n+1}, y_n, t) \geq 1, \ \forall\ n \in \mathcal{N} \ and \ \forall\ t > 0 \tag{2.2}$$

From(2.1.1), we have
$x_{2n+1} \preceq fx_{2n+1} = Tx_{2n+2} \preceq fTx_{2n+2} \preceq x_{2n+2}$,
$x_{2n+2} \preceq gx_{2n+2} = Sx_{2n+3} \preceq gSx_{2n+3} \preceq x_{2n+3}$. Thus

$$x_n \preceq x_{n+1}, \forall n \in \mathcal{N} \tag{2.3}$$

Case (i): Suppose $y_{2m} = y_{2m+1}$ for some m.
Assume that $y_{2m+1} \neq y_{2m+2}$.
Then there exists $t_0 > 0$ such that $0 < M(y_{2m+1}, y_{2m+2}, t_0) < 1$.
From (1), $\alpha(Sx_{2m+1}, Tx_{2m+2}, t_0) = \alpha(y_{2m}, y_{2m+1}, t_0) \geq 1$.
Now from (3) and (2.1.3), we have

$$\begin{aligned}
\psi\left(\frac{1}{M(y_{2m+1}, y_{2m+2}, t_0)} - 1\right) \quad &= \psi\left(\frac{1}{M(fx_{2m+1}, gx_{2m+2}, t_0)} - 1\right) \\
&\leq \alpha(Sx_{2m+1}, Tx_{2m+2}, t_0)\psi\left(\frac{1}{M(fx_{2m+1}, gx_{2m+2}, t_0)} - 1\right) \\
&\leq \phi\left(\frac{1}{m(x_{2m+1}, x_{2m+2}, t_0)} - 1\right) - \varphi\left(\frac{1}{m(x_{2m+1}, x_{2m+2}, t_0)} - 1\right)
\end{aligned}$$

where

$$m(x_{2m+1}, x_{2m+2}, t_0) \;\; = \min\{M(y_{2m+1}, y_{2m}, t_0), M(y_{2m+1}, y_{2m}, t_0), M(y_{2m+2}, y_{2m+1}, t_0)\} \\ = M(y_{2m+1}, y_{2m+2}, t_0).$$

Thus
$\psi\left(\frac{1}{M(y_{2m+1},y_{2m+2},t_0)} - 1\right) \leq \phi\left(\frac{1}{M(y_{2m+1},y_{2m+2},t_0)} - 1\right) - \varphi\left(\frac{1}{M(y_{2m+1},y_{2m+2},t_0)} - 1\right)$.
It is a contradiction to (A). Hence $y_{2m+1} = y_{2m+2}$.
Continuing in this way, we get $y_{2m} = y_{2m+1} = y_{2m+2} = \cdots$
Hence $\{y_n\}$ is a Cauchy sequence in X.
Case (ii): Assume that $y_n \neq y_{n+1}$,$\forall n$.
As in Case (i), we have

$$\psi\left(\frac{1}{M(y_{2n+1},y_{2n+2},t)} - 1\right) \;\; \leq \phi\left(\frac{1}{m(x_{2n+1},x_{2n+2},t)} - 1\right) - \varphi\left(\frac{1}{m(x_{2n+1},x_{2n+1},t)} - 1\right)$$

where

$$m(x_{2n+1}, x_{2n+2}, t) = \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)\}.$$

If $m(x_{2n+1}, x_{2n+2}, t) = M(y_{2n+1}, y_{2n+2}, t)$ then
$\psi\left(\frac{1}{M(y_{2n+1},y_{2n+2},t)} - 1\right) \leq \phi\left(\frac{1}{M(y_{2n+1},y_{2n+2},t)} - 1\right) - \varphi\left(\frac{1}{M(y_{2n+1},y_{2n+2},t)} - 1\right)$.
It is a contradiction to (A). Hence

$$\psi\left(\frac{1}{M(y_{2n+1},y_{2n+2},t)} - 1\right) \;\; \leq \phi\left(\frac{1}{M(y_{2n},y_{2n+1},t)} - 1\right) - \varphi\left(\frac{1}{M(y_{2n},y_{2n+1},t)} - 1\right) \\ < \psi\left(\frac{1}{M(y_{2n},y_{2n+1},t)} - 1\right), \text{ from } (A). \tag{2.4}$$

Since ψ is monotonically increasing we have

$$M(y_{2n+1}, y_{2n+2}, t) \geq M(y_{2n}, y_{2n+1}, t), \;\forall\; t > 0.$$

Similarly by using (2) and proceding as above we can show that

$$M(y_{2n+2}, y_{2n+3}, t) \geq M(y_{2n+1}, y_{2n+2}, t), \;\forall\; t > 0.$$

Thus $M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t)$ for $n = 2, 3, \cdots$ and $\forall t > 0$.
Thus $\{M(y_n, y_{n+1}, t)\}$ is an increasing sequence of positive real numbers in $[0, 1]$ and hence converges to some $r(t)$,$\forall t > 0$.
Thus $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = r(t), \forall t > 0$.
Suppose there exists some $t_0 > 0$ such that $r(t_0) < 1$.
Letting $n \to \infty$ in (2.4) and using continuity, upper semi continuity and lower semi continuity of ψ, ϕ and φ respectively, we get

$$\psi\left(\frac{1}{r(t_0)} - 1\right) \leq \phi\left(\frac{1}{r(t_0)} - 1\right) - \varphi\left(\frac{1}{r(t_0)} - 1\right).$$

It is a contradiction from (A).
Hence $r(t) = 1$,$\forall t > 0$.
Thus

$$\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1, \;\forall\; t > 0. \tag{2.5}$$

Now for each positive integer p, we have

$$M(y_n, y_{n+p}, t) \geq M\left(y_n, y_{n+1}, \frac{t}{p}\right) * M\left(y_{n+1}, y_{n+2}, \frac{t}{p}\right) * \cdots * M\left(y_{n+p-1}, y_{n+p}, \frac{t}{p}\right).$$

letting $n \to \infty$ and using (5), we get

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) = 1, \ \forall\, t > 0.$$

Hence $\{y_n\}$ is a G-Cauchy sequence in X.
Since X is G-complete, there exists $z \in X$ such that $\{y_n\}$ converges to z. Thus $\lim_{n\to\infty} M(y_n, z, t) = 1, \ \forall\, t > 0$. Hence

$$\lim_{n\to\infty} fx_{2n+1} = \lim_{n\to\infty} gx_{2n+2} = \lim_{n\to\infty} Tx_{2n+2} = \lim_{n\to\infty} Sx_{2n+1} = z.$$

Suppose $(2.1.6)(a)$ holds.
Since S is continuous, we have $S^2x_{2n+1} \to Sz$ and $Sfx_{2n+1} \to Sz$.
Since the pair (f, S) is compatible, we have

$$\lim_{n\to\infty} M(fSx_{2n+1}, Sfx_{2n+1}, t) = 1, \ \forall\, t > 0.$$

Hence $fSx_{2n+1} \to Sz$.
Now from (2.1.6)(a), we have

$$\alpha(SSx_{2n+1}, Tx_{2n}, t) = \alpha(Sy_{2n}, y_{2n-1}, t) \geq 1.$$

From(2.1.1), we have $x_{2n} \preceq gx_{2n} = Sx_{2n+1}$.
By using (2.1.3), we have

$$\begin{aligned}\psi\left(\tfrac{1}{M(fSx_{2n+1}, gx_{2n}, t)} - 1\right) &\leq \alpha(SSx_{2n+1}, Tx_{2n}, t)\psi\left(\tfrac{1}{M(fSx_{2n+1}, gx_{2n}, t)} - 1\right)\\ &\leq \phi\left(\tfrac{1}{m(Sx_{2n+1}, x_{2n}, t)} - 1\right) - \varphi\left(\tfrac{1}{m(Sx_{2n+1}, x_{2n}, t)} - 1\right)\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}m(Sx_{2n+1}, x_{2n}, t) &= \min\{M(SS_{2n+1}, Tx_{2n}, t), M(SS_{2n+1}, fSx_{2n+1}, t), M(Tx_{2n}, gx_{2n}, t)\}\\ &\to M(Sz, z, t) \text{ as } n \to \infty.\end{aligned}$$

Letting $n \to \infty$ in (6), we get

$$\psi\left(\frac{1}{M(Sz, z, t)} - 1\right) \leq \phi\left(\frac{1}{M(Sz, z, t)} - 1\right) - \varphi\left(\frac{1}{M(Sz, z, t)} - 1\right)$$

which in turn yields from (A) that $Sz = z$.
Since $x_{2n} \preceq gx_{2n}$ and $gx_{2n} \to z$,by (2.1.7), we have $x_{2n} \preceq z$.
From (2.1.6)(a), we have $\alpha(Sz, Tx_{2n}, t) = \alpha(z, y_{2n-1}, t) \geq 1$.
By using (2.1.3), we have

$$\begin{aligned}\psi\left(\tfrac{1}{M(fz, gx_{2n}, t)} - 1\right) &\leq \alpha(Sz, Tx_{2n}, t)\psi\left(\tfrac{1}{M(fz, gx_{2n}, t)} - 1\right)\\ &\leq \phi\left(\tfrac{1}{m(z, x_{2n}, t)} - 1\right) - \varphi\left(\tfrac{1}{m(z, x_{2n}, t)} - 1\right)\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}m(z, x_{2n}, t) &= \min\{M(Sz, Tx_{2n}, t), M(Sz, fz, t), M(Tx_{2n}, gx_{2n}, t)\}\\ &\to M(z, fz, t) \text{ as } n \to \infty.\end{aligned}$$

Letting $n \to \infty$ in (7), we get

$$\psi\left(\frac{1}{M(fz,z,t)}-1\right) \leq \phi\left(\frac{1}{M(fz,z,t)}-1\right) - \varphi\left(\frac{1}{M(fz,z,t)}-1\right)$$

which in turn yields from (A) that $fz = z$.
Since $f(X) \subseteq T(X)$, there exists $w \in X$ such that $z = fz = Tw$. Also we have $z = fz = Tw \preceq fTw \preceq w$.
From(2.1.6)(a),$\alpha(Sz, Tw, t) = \alpha(z, z, t) \geq 1$.
By using (2.1.3), we have

$$\begin{aligned} \psi\left(\frac{1}{M(Tw,gw,t)}-1\right) &= \psi(\left(\frac{1}{M(fz,gw,t)}-1\right) \\ &\leq \alpha(Sz,Tw,t)\psi\left(\frac{1}{M(fz,gw,t)}-1\right) \\ &\leq \phi\left(\frac{1}{m(z,w,t)}-1\right) - \varphi\left(\frac{1}{m(z,w,t)}-1\right) \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} m(z,w,t) &= \min\{M(Sz,Tw,t), M(fz,Sz,t), M(gw,Tw,t)\} \\ &\to M(gw,Tw,t). \end{aligned}$$

Thus

$$\psi\left(\frac{1}{M(Tw,gw,t)}-1\right) \leq \phi\left(\frac{1}{M(Tw,gw,t)}-1\right) - \varphi\left(\frac{1}{M(Tw,gw,t)}-1\right)$$

which in turn yields from (A) that $gw = Tw = z$.
Since the pair(g, T) is weakly compatible , we have $gz = gTw = Tgw = Tz$.
Since $x_{2n+1} \preceq fx_{2n+1}$ and $fx_{2n+1} \to z$, by (2.1.7), we have $x_{2n+1} \preceq z$.
From(2.1.6)(a), $\alpha(Sx_{2n+1}, Tz, t) = \alpha(y_{2n}, Tz, t) \geq 1$.
From(2.1.3),we have

$$\begin{aligned} \psi\left(\frac{1}{M(fx_{2n+1},gz,t)}-1\right) &\leq \alpha(Sx_{2n+1},Tz,t)\psi\left(\frac{1}{M(fx_{2n+1},gz,t)}-1\right) \\ &\leq \phi\left(\frac{1}{m(x_{2n+1},z,t)}-1\right) - \varphi\left(\frac{1}{m(x_{2n+1},z,t)}-1\right) \end{aligned} \tag{2.9}$$

where

$$\begin{aligned} m(x_{2n+1},z,t) &= \min\{M(y_{2n},Tz,t), M(y_{2n+1},y_{2n},t), M(gz,Tz,t)\} \\ &\to M(z,gz,t) \text{ as } n \to \infty. \end{aligned}$$

Letting $n \to \infty$ in (9), we get

$$\psi\left(\frac{1}{M(z,gz,t)}-1\right) \leq \phi\left(\frac{1}{M(z,gz,t)}-1\right) - \varphi\left(\frac{1}{M(z,gz,t)}-1\right)$$

which in turn yields from (A) that $gz = z$. Hence $Tz = z$.
Thus z is a common fixed point of f, g, S and T.
Uniqueness of common fixed point follows easily by (2.1.8).
Similarly we can prove the theorem when (2.1.6)(b) holds.
Now we give an example to illustrate Theorem 2.1

Example 2.2. Let $X = [0, \infty)$and define $x \preceq y$ if $y \leq x$. Put $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. For all $x, y \in X$ define $M(x, y, t) = \frac{t}{t+|x-y|}$for $t > 0$ and $M(x, y, 0) = 0$. Define $f, g, S, T : X \to X$ by $fx = \frac{x}{2}, gx = \frac{x}{4}, Sx = 8x$ and $Tx = 4x$ for all $x \in X$.

Let $\psi, \phi, \varphi : \mathcal{R}^+ \to \mathcal{R}^+$ be defined as $\psi(t) = t, \phi(t) = \frac{3t}{4}$ and $\varphi(t) = \frac{t}{4}$.

Define $\alpha(x, y, t) = \begin{cases} 1, & \text{if } x, y \in [0, 1], \\ 0, & \text{otherwise} \end{cases}$ for all $t > 0$.

We have $fx = \frac{x}{2} \leq x \Rightarrow x \preceq fx$ and $gx = \frac{x}{4} \leq x \Rightarrow x \preceq gx$.
Also $fTx = 2x \geq x \Rightarrow fTx \preceq x$ and $gSx = 2x \geq x \Rightarrow gSx \preceq x$.
If $x > \frac{1}{8}$ and $y \in X$ then $\alpha(Sx, Ty) = 0$.
If $x \leq \frac{1}{8}$ and $y > \frac{1}{4}$ then $\alpha(Sx, Ty) = 0$.
In these cases,the condition (2.1.3) is clearly satisfied.
Suppose $x \leq \frac{1}{8}$ and $y \in [0, \frac{1}{4}]$ then $\alpha(Sx, Ty) = 1$.
We have $\frac{1}{M(fx,gy,t)} - 1 = \frac{|2x-y|}{4t}$ and $\frac{1}{M(Sx,Ty,t)} - 1 = \frac{4\ |2x-y|}{t}$.
Clearly $\frac{1}{m(x,y,t)} - 1 \geq \frac{1}{M(Sx,Ty,t)} - 1$ for all $x, y \in X$ and for all $t > 0$.
Now,

$$\begin{aligned}
\phi(\frac{1}{m(x,y,t)} - 1) - \varphi(\frac{1}{m(x,y,t)} - 1) &= \frac{1}{2}(\frac{1}{m(x,y,t)} - 1) \\
&\geq \frac{1}{2}(\frac{1}{M(Sx,Ty,t)} - 1) \\
&= \frac{2\ |2x-y|}{t} \\
&= 8\ (\frac{1}{M(fx,gy,t)} - 1) \\
&> \psi(\frac{1}{M(fx,gy,t)} - 1) \\
&= \alpha(Sx, Ty, t)\psi(\frac{1}{M(fx,gy,t)} - 1)
\end{aligned}$$

Thus (2.1.3) is satisfied.
One can easily verify all the other conditions of Theorem 2.1. Clearly 0 is the unique common fixed point of f, g, S and T.
By suitably taking α, ψ, ϕ and φ in Theorem 2.1,one can obtain some previous results in fuzzy metric spaces.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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