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Infinitesimal CL-transformations on Kenmotsu manifolds

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Abstract The present paper deals with the study of infinitesimal CL-transformations on Kenmotsu manifolds, whose metric tensor is a Ricci soliton. We obtain the conditions that the Ricci solitons to be expanding, steady and shrinking. Among others we find a necessary and sufficient condition of a Ricci soliton on Kenmotsu manifold with respect to CL-connection to be Ricci soliton on Kenmotsu manifold with respect to Levi-Civita connection.

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1. Introduction

In [26] Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He proved that they could be divided into three classes: (i) homogeneous normal contact Riemannian manifolds with $c > 0$, (ii) global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$ and (iii) a warped product space if $c < 0$. It is known that the manifolds of class (i) are characterized by admitting a Sasakian structure. The manifolds of class (ii) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [15] characterized the differential geometric properties of the manifolds of class (iii) which are nowadays called Kenmotsu manifolds and later studied by several authors.

As a generalization of both Sasakian and Kenmotsu manifolds, Oubiña [18] introduced the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kähler manifolds. A trans-Sasakian manifold of type (0, 0), $(\alpha, 0)$ and $(0, \beta)$ are called

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the cosympletic, α-Sasakian and β-Kenmotsu manifolds respectively, α, β being scalar functions. In particular, if $\alpha = 0, \beta = 1$; and $\alpha = 1, \beta = 0$ then a trans-Sasakian manifold will be a Kenmotsu and Sasakian manifold respectively.

It is known that loxodrome is a curve on the unit sphere that intersects the meridians at a fixed angle and C-loxodrome is a loxodrome cutting geodesic trajectories of the characteristic vector field ξ of the Sasakian manifold with constant angle. In 1963, Tashiro and Tachibana [25] introduced a transformation, called CL-transformation, on a sasakian manifold under which C-loxodrome remains invariant. Here 'CL' stands for C-loxodrome. CL-transformation have been studied by various authors in different context such as Koto and Nagao [16], Takamatsu and Mizusawa [24], Shaikh et. al ([21], [22]) and many others.

In 1982, Hamilton [11] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. Then Ricci flow has become a powerful tool for the study of Riemannian manifolds, especially for those manifolds with positive curvature. Perelman ([19], [20]) used Ricci flow and its surgery to prove Poincare conjecture. The Ricci flow is an evolution equation for metrics on a Riemannian manifold defined as follows:

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij}.$$

A Ricci soliton emerges as the limit of the solutions of the Ricci flow. A solution to the Ricci flow is called Ricci soliton if it moves only by a one parameter group of diffeomorphism and scaling. A Ricci soliton (g, V, λ) on a Riemannian manifold (M, g) is a generalization of an Einstein metric such that [12]

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1.1}$$

where S is the Ricci tensor, $\pounds_V$ is the Lie derivative operator along the vector field V on M and λ is a real number. The Ricci soliton is said to be shrinking, steady and expanding according as λ is negative, zero and positive respectively.

During the last two decades, the geometry of Ricci solitons has been the focus of attention of many mathematicians. In particular, it has become more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904. In [23] Sharma studied the Ricci solitons in contact geometry. Thereafter Ricci solitons in contact metric manifolds have been studied by various authors such as Bagewadi et. al ([1], [2], [3], [14]), Bejan and Crasmareanu [4], Blaga [6], Chandra et. al [7], Chen and Deshmukh [8], Deshmukh et. al [10], He and Zhu [13], Nagaraja and Premalatta [17], Tripathi [27] and many others.

Motivated by the above studies the present paper deals with the study of infinitesimal CL-transformations on Kenmotsu manifolds whose metric is Ricci soliton. The paper is organized as follows. Section 2 is concerned with preliminaries. Section 3 is devoted to the study of infinitesimal CL-transformations and Ricci solitons on Kenmotsu manifolds. It is proved that if (g, V, λ) is a Ricci soliton on a Kenmotsu manifold M such that V is an infinitesimal CL-transformation, then V is a projective killing vector field. In [16] Koto and Nagao introduced a new type of an affine connection, called CL-connection. In this section we study Ricci solitons on Kenmotsu manifolds with respect to CL-connection and obtain a necessary and sufficient condition of a Ricci soliton on Kenmotsu manifold with respect to CL-connection to be a Ricci soliton on Kenmotsu manifold with respect to Levi-Civita connection. Among others Ricci soliton on CL-flat (respectively CL-symmetric and CL-semisymmetric) Kenmotsu manifolds are also investigated.

2. Preliminaries

A smooth manifold (M^n, g) $(n = 2m + 1 > 3)$ is said to be an almost contact metric manifold [5] if it admits a (1,1) tensor field ϕ, a vector field ξ, an 1-form η and a Riemannian metric g which satisfy

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \qquad \phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, Y) = -g(X, \phi Y), \quad \eta(X) = g(X, \xi), \quad \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for all vector fields X, Y on M.

An almost contact metric manifold $M^n(\phi, \xi, \eta, g)$ is said to be Kenmotsu manifold if the following condition holds [15]:

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.5}$$

where ∇ denotes the Riemannian connection of g.
In a Kenmotsu manifold $M^n(\phi, \xi, \eta, g)$, the following relations hold [15]:

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.6}$$

$$S(X, \xi) = -(n-1)\eta(X) \tag{2.7}$$

for any vector field X, Y, Z on M and R is the Riemannian curvature tensor and S is the Ricci tensor of type (0,2) such that $g(QX, Y) = S(X, Y)$.

3. Infinitesimal CL-trasformations and Ricci solitons

This section deals with the infinitesimal CL-trasformations on Kenmotsu manifolds whose metric tensor is Ricci soliton.

Definition 3.1. A vector field V on a Kenmotsu manifold M is said to be an infinitesimal CL-transformation ([21], [24]) if it satisfies

$$\pounds_V \{^{\,h}_{ij}\} = \rho_j \delta^h_i + \rho_i \delta^h_j + \alpha(\eta_j \phi^h_i + \eta_i \phi^h_j) \tag{3.1}$$

for a certain constant α, where ρ_i are the components of the 1-form ρ, $\pounds_V$ denotes the Lie derivative with respect to V and $\{^{\,h}_{ij}\}$ is the Christoffel symbol of the Riemannian metric g.

In [21], Shaikh et. al studied infinitesimal CL-transformations on a Kenmotsu manifold M and obtained the following useful result:

Proposition 3.2. *[21] If V is an infinitesimal CL-transformations on a Kenmotsu manifold M, then the 1-form ρ is closed.*

Theorem 3.3. *[21] If V is an infinitesimal CL-transformation on a Kenmotsu manifold M, then the relation*

$$(\pounds_V g)(Y,Z) = (\nabla_Y \rho)(Z) - \alpha g(Y,\phi Z) \tag{3.2}$$

holds for any vector fields Y and Z on M.

From (1.1) and (3.2), we obtain

$$S(Y,Z) = -\lambda g(Y,Z) + \frac{\alpha}{2} g(Y,\phi Z) - \frac{1}{2}(\nabla_Y \rho)(Z). \tag{3.3}$$

Since the Ricci tensor S and the metric tensor g are symmetric and the 1-form ρ is closed by Proposition 3.1, so interchanging Y and Z in (3.3) and subtracting the obtained result from (3.3), we get by virtue of (2.2) that

$$\alpha g(Y,\phi Z) = 0,$$

which implies that $\alpha = 0$ and hence the infinitesimal CL-transformation V is a projective killing vector field. Also (3.3) yields

$$S(Y,Z) = -\lambda g(Y,Z) - \frac{1}{2}(\nabla_Y \rho)(Z). \tag{3.4}$$

This leads to the following:

Theorem 3.4. *If (g,V,λ) is a Ricci soliton on a Kenmotsu manifold M such that V is an infinitesimal CL-transformation, then V is a projective killing vector field and the Ricci tensor S is given by (3.4).*

Definition 3.5. [16] A transformation f on a $n(=2m+1)$-dimensional Kenmotsu manifold M with structure (ϕ,ξ,η,g) is said to be a CL-transformation if the Levi-Civita connection ∇ and a symmetric affine connection ∇^f, called CL-connection, induced from ∇ by f are related by

$$\nabla^f_X Y = \nabla_X Y + \rho(X)Y + \rho(Y)X + \alpha\{\eta(X)\phi Y + \eta(Y)\phi X\}, \tag{3.5}$$

where ρ is an 1-form and α is a constant.

If R and R^f are respectively the curvature tensor with respect to Levi-Civita connection ∇ and CL-connection ∇^f in a Kenmotsu manifold then we have [21]

$$\begin{aligned} R^f(X,Y)Z &= R(X,Y)Z + B(X,Z)Y - B(Y,Z)X \\ &- \alpha\Big[\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) \\ &+ \{g(Y,Z)\phi X - g(X,Z)\phi Y\} - \{g(Y,\phi Z)\eta(X) \\ &- g(X,\phi Z)\eta(Y) - 2g(X,\phi Y)\eta(Z)\}\xi\Big] \end{aligned} \tag{3.6}$$

for all vector fields X, Y, Z on M, where the symmetric tensor field B is given by

$$\begin{aligned} B(X,Y) &= (\nabla_X \rho)(Y) - \rho(X)\rho(Y) - \alpha^2\eta(X)\eta(Y) \\ &- \alpha\big[\eta(X)\rho(\phi Y) + \eta(Y)\rho(\phi X)\big]. \end{aligned} \tag{3.7}$$

From (3.6) we get

$$S^f(Y,Z) = S(Y,Z) - (n-1)B(Y,Z), \tag{3.8}$$

where S^f and S are respectively the Ricci tensor of a Kenmotsu manifold with respect to the CL-connection ∇^f and Levi-Civita connection ∇.

We now consider (g, V, λ) is a Ricci soliton on a Kenmotsu manifold M with respect to CL-connection ∇^f. Then we have

$$(\pounds^f_V g)(Y, Z) + 2S^f(Y, Z) + 2\lambda g(Y, Z) = 0, \tag{3.9}$$

where $\pounds^f_V$ is the Lie derivative along the vector field V on M with respect to CL-connection ∇^f.
By virtue of (3.5) we have

$$\begin{aligned} & (\pounds^f_V g)(Y, Z) \\ = \; & g(\nabla^f_Y V, Z) + g(Y, \nabla^f_Z V) \\ = \; & g\big(\nabla_Y V + \rho(Y)V + \rho(V)Y + \alpha\{\eta(Y)\phi V + \eta(V)\phi Y\}, Z\big) \\ + \; & g\big(Y, \nabla_Z V + \rho(Z)V + \rho(V)Z + \alpha\{\eta(Z)\phi V + \eta(V)\phi Z\}\big) \\ = \; & (\pounds_V g)(Y, Z) + \rho(Y)g(V, Z) + \rho(Z)g(Y, V) \\ + \; & 2\rho(V)g(Y, Z) + \alpha\{\eta(Y)g(\phi V, Z) + \eta(Z)g(Y, \phi V)\}. \end{aligned} \tag{3.10}$$

In view of (3.8) and (3.10), (3.9) yields

$$\begin{aligned} & (\pounds_V g)(Y, Z) + 2S(Y, Z) + 2\lambda g(Y, Z) \\ + \; & \rho(Y)g(V, Z) + \rho(Z)g(Y, V) + 2\rho(V)g(Y, Z) \\ + \; & \alpha\{\eta(Y)g(\phi V, Z) + \eta(Z)g(Y, \phi V)\} - 2(n-1)B(Y, Z) = 0. \end{aligned} \tag{3.11}$$

If (g, V, λ) is a Ricci soliton on a Kenmotsu manifold with respect to Levi-Civita connection then (1.1) holds. Thus from (1.1) and (3.11) we can state the following:

Theorem 3.6. *A Ricci soliton (g, V, λ) on a Kenmotsu manifold is invariant under CL-connection if and only if the relation*

$$\begin{aligned} & \rho(Y)g(V, Z) + \rho(Z)g(Y, V) + 2\rho(V)g(Y, Z) \\ + \; & \alpha\{\eta(Y)g(\phi V, Z) + \eta(Z)g(Y, \phi V)\} - 2(n-1)B(Y, Z) = 0 \end{aligned}$$

holds for arbitrary vector fields Y, Z and V.

Now, let (g, ξ, λ) be a Ricci soliton on a Kenmotsu manifold with respect to CL-connection. Then we have

$$(\pounds^f_\xi g)(Y, Z) + 2S^f(Y, Z) + 2\lambda g(Y, Z) = 0. \tag{3.12}$$

From (2.1), (2.2), (2.4) and (3.5) we have

$$\begin{aligned} & (\pounds^f_\xi g)(Y, Z) \\ = \; & g(\nabla^f_Y \xi, Z) + g(Y, \nabla^f_Z \xi) \\ = \; & g(Y - \eta(Y)\xi + \rho(Y)\xi + \rho(\xi)Y + \alpha\phi Y, Z) \\ + \; & g(Y, Z - \eta(Z)\xi + \rho(Z)\xi + \rho(\xi)Z + \alpha\phi Z) \\ = \; & 2[\{1 + \rho(\xi)\}g(Y, Z) - \eta(Y)\eta(Z)] + \rho(Y)\eta(Z) + \rho(Z)\eta(Y). \end{aligned} \tag{3.13}$$

Using (3.8) and (3.13) in (3.12), we get

$$\begin{aligned} S(Y, Z) \; = \; & -\{\lambda + 1 + \rho(\xi)\}g(Y, Z) + \eta(Y)\eta(Z) \\ + \; & (n-1)B(Y, Z) - \frac{1}{2}\{\rho(Y)\eta(Z) + \rho(Z)\eta(Y)\}. \end{aligned} \tag{3.14}$$

This leads to the following:

Theorem 3.7. *If (g, ξ, λ) is a Ricci soliton on a Kenmotsu manifold M with respect to CL-connection then the Ricci tensor S is given by (3.14).*

Putting $Z = \xi$ in (3.14) and using (2.1) and (2.2), we get

$$S(Y, \xi) = -\{\lambda + \frac{3}{2}\rho(\xi)\}\eta(Y) + (n-1)B(Y, \xi) - \frac{1}{2}\rho(Y). \tag{3.15}$$

From (2.7) and (3.15), we obtain

$$-\{\lambda + \frac{3}{2}\rho(\xi)\}\eta(Y) + (n-1)B(Y, \xi) - \frac{1}{2}\rho(Y) = -(n-1)\eta(Y). \tag{3.16}$$

Setting $Y = \xi$ in (3.16) we get

$$\lambda = (n-1)[1 + B(\xi, \xi)] - 2\rho(\xi). \tag{3.17}$$

This leads to the following:

Theorem 3.8. *A Ricci soliton (g, ξ, λ) on a Kenmotsu manifold $M^n(\phi, \xi, \eta, g)$ with respect to CL-connection is shrinking, steady and expanding according as $(n-1)[1 + B(\xi, \xi)] - 2\rho(\xi) < 0$, $(n-1)[1 + B(\xi, \xi)] = 2\rho(\xi)$ and $(n-1)[1 + B(\xi, \xi)] - 2\rho(\xi) > 0$ respectively.*

Also, Shaikh et. al [21] proved the tensor field

$$\begin{aligned} A(X,Y)Z &= R(X,Y)Z - \frac{1}{n-1}\Big[\{S(Y,Z)X - S(X,Z)Y\} \\ &- \{g(Y,Z) + \eta(Y)\eta(Z)\}QX + \{g(X,Z) + \eta(X)\eta(Z)\}QY \\ &+ \{S(X,Z) + (n-1)g(X,Z)\}\eta(Y)\xi \\ &- \{S(Y,Z) + (n-1)g(Y,Z)\}\eta(X)\xi \\ &+ 2\{S(X,Y) + (n-1)g(X,Y)\}\eta(Z)\xi\Big] \\ &+ \{g(Y,Z) + \eta(Y)\eta(Z)\}X - \{g(X,Z) + \eta(X)\eta(Z)\}Y \end{aligned}$$

is invariant on a Kenmotsu manifold M under a CL-transformation, and it is called the CL-curvature tensor field on M.

Definition 3.9. [21] A Kenmotsu manifold M is said to be CL-flat if the CL-curvature tensor field A of the type $(1, 3)$ vanishes identically on M.

Definition 3.10. [21] A Kenmotsu manifold M is said to be CL-symmetric if $\nabla A = 0$.

Definition 3.11. [21] A Kenmotsu manifold M is said to be CL-semisymmetric if the $R(X, Y) \cdot A = 0$.

In [21], Shaikh et. al proved that in a Kenmotsu manifold M, the concept of CL-semisymmetry, CL-symmetry, CL-flatness and manifold of constant curvature -1, i.e. manifold is Einsteinnian are equivalent and its Ricci tensor is of the form

$$S(Y, Z) = -(n-1)g(Y, Z). \tag{3.18}$$

Again in [9], Debnath and Bhattacharyya studied second order parallel tensor in trans-Sasakian manifolds and as a corollary of their result we have the following:

Theorem 3.12. *In a Kenmotsu manifold M, every second order parallel symmetric tensor is a constant multiple of the metric tensor.*

Suppose that the (0,2) type symmetric tensor field $\pounds_V g + 2S$ is parallel for any vector field V on a Kenmotsu manifold M. Then Theorem 3.6 yields $\pounds_V g + 2S$ is a constant multiple of the metric tensor g, i.e. $(\pounds_V g)(X,Y) + 2S(X,Y) = -2\lambda g(X,Y)$ for all X, Y on M, where λ is a constant. Hence the relation (1.1) holds. This implies that (g,V,λ) yields a Ricci soliton. Hence we can state the following:

Theorem 3.13. *If the tensor field $\pounds_V g + 2S$ on a Kenmotsu manifold is parallel for any vector field V, then (g,V,λ) is a Ricci soliton.*

Let us consider h be a $(0,2)$ symmetric parallel tensor field on a Kenmotsu manifold such that

$$h(X,Y) = (\pounds_\xi g)(X,Y) + 2S(X,Y). \tag{3.19}$$

From (2.4) we have

$$(\pounds_\xi g)(X,Y) = g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi) = 2[g(X,Y) - \eta(X)\eta(Y)]. \tag{3.20}$$

Using (3.18) and (3.20) in (3.19), we get

$$h(X,Y) = -2(n-2)g(X,Y) - 2\eta(X)\eta(Y). \tag{3.21}$$

Putting $X = Y = \xi$ in (3.21), we obtain

$$h(\xi,\xi) = -2(n-1). \tag{3.22}$$

If (g,ξ,λ) is a Ricci soliton on a Kenmotsu manifold M, then from (1.1) we have

$$h(X,Y) = -2\lambda g(X,Y) \tag{3.23}$$

and hence

$$h(\xi,\xi) = -2\lambda. \tag{3.24}$$

From (3.22) and (3.24) we get $\lambda = (n-1) > 0$ and consequently the Ricci soliton (g,ξ,λ) is expanding. Thus we can state the following:

Theorem 3.14. *If the tensor field $\pounds_\xi g + 2S$ on a CL-flat (respectively CL-symmetric, CL-semisymmetric) Kenmotsu manifold is parallel, then the Ricci soliton (g,ξ,λ) is always expanding.*

Conflict of Interests

The author declare that there is no conflict of interests regarding the publication of this paper.

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